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Microstructure theory applied in RMB exchange rate and Bitcoin market price

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Microstructure theory applied in RMB exchange rate and Bitcoin market price

by

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The student author, whose presentation of the scholarship herein was approved by the program of study committee, is solely responsible for the content of this dissertation. The Graduate College will ensure this dissertation is globally accessible and will not permit alterations after a degree is conferred.

Iowa State University

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DEDICATION

In dedication to my beloved parents, my wife Ge Guo, and my son Salomon Wu.

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ABSTRACT

This dissertation presents the application of microstructure theory on the RMB exchange rate and Bitcoin market price. The existing research on the RMB exchange rate and Bitcoin market price mainly studied their statistical characteristics through empirical methodologies. This dissertation fills the research gap in microstructure theory applied to the RMB exchange rate and Bitcoin market price. First, the model for the determination of the two Renminbi (RMB) exchange rates and their interactions is established, and empirical analysis suggests that the interactions among the two exchange rates and the explanatory variables are time-varying, in particular, after the "811 RMB exchange rate reform", the offshore RMB exchange rate replaced the onshore RMB exchange rate as the leading indicator. Second, a model describing the speculative behavior in the Bitcoin trading market is developed. This theoretical model captures the statistical characteristics of Bitcoin market prices. The fundamental value of Bitcoin system is controversial, and the mysterious and innovative features of the Bitcoin system incite the speculation behaviours. The speculation leads to the market bubble that brought the soaring and plunges of Bitcoin market price. Finally, an economic model for Bitcoin mining competition based on the Bitcoin protocol is established, which provides a benchmark for further research on mining competition in economics. For any Bitcoin miners, the equilibrium input depends on the comparison of the miner's own marginal cost with that of other miners, however, whether profit can be obtained or not depends on the miner's own fixed cost.

CHAPTER 1. GENERAL INTRODUCTION

A typical fact that is widely documented in the empirical literature on exchange rate determination is that the macro-economic fundamentals are the determinants of exchange rate over a long period of time, while a large amount of fluctuations in spot exchange rate are observed without significant changes in the macro-economic fundamentals. The microstructure literature attempts to illustrate the mechanisms that lead to the deviations of spot exchange rate from the macro-economic fundamentals.

The model assumptions and methodologies of microstructure theory are very different from that of the traditional macro-economic theory. Regarding the model assumptions, the traditional macro-economic theory suggests that public information exclusively determines the exchange rate. In addition, the agents in the foreign exchange market are homogeneous, and the influence of market trading rule is ignored. Meanwhile, the microstructure theory focuses on the issues that are not incorporated by the traditional macro-economic theory, such as the information transferring among the market participants, the heterogeneity of market agent expectations, and how the heterogeneity cause the volatility of exchange rate. On the other side, regarding the methodology, the determination of the exchange rate in the traditional macro-economic theory depends on the correlations among a series of macro-economic variables. However, in microstructure theory, the exchange rate is determined by the interactions among various decision-making agents in the foreign exchange market.

The offshore Renminbi (RMB) exchange rate with the Hong Kong foreign exchange market as the trading center is the symbol of RMB internationalization. Since the official establishment of the offshore RMB exchange rate in 2012, two different exchange rates of the same currency (RMB) co-exist in the market. One is the managed-floating exchange rate of RMB determined in the onshore inter-bank foreign exchange market, the other one is the free-floating exchange rate of RMB determined in the Hong Kong market. Most of the previous literature on the interactions of two RMB exchange rates focus on the time-varying relationship by empirical methodologies. However, there is a lack of discussion on how the two RMB exchange rates are determined in the two different markets separately, and how they interact with each other. Therefore, the first paper builds a model on the determination of both onshore and offshore RMB exchange rates. Firstly, given the onshore RMB exchange rate, how do the offshore foreign exchange market participants determine the offshore RMB exchange rate? Secondly, in the onshore inter-bank foreign exchange market, how do the commercial banks maximize their value of foreign exchange transactions and how does the People's Bank of China (PBoC), working as a heavy interventionist, manage the onshore exchange rate in order to smooth the fluctuations? Thirdly, the PBoC re-ignited the reform of onshore exchange rate pricing mechanism on August 11^{th}, 2015, what are the statistical characteristics of the two RMB exchange rates before and after the exchange rate reform?

The effective exchange rate of RMB is exogenous in the model of two RMB exchange rates determination in the first paper. In other words, the fundamental value of the onshore and offshore RMB exchange rates is exogenously determined. In the second paper, the model of asset pricing mechanism of which the fundamental value evolves with time is constructed for Bitcoin. The Bitcoin system is an innovative payment technology with the advantages of un-retroactivity and anonymousness. However, the proof-of-working mechanism leads to the transaction congestion in Bitcoin system. Therefore, Bitcoin system requires further improvement and the measurement of its fundamental value remains controversial.

Besides, the market price of Bitcoin has experienced several soaring and plunges after its debut in 2008. The previous literature on Bitcoin market price primarily measures the volatility of market price, while there is a lack of analysis on the measurement of the fundamental value of Bitcoin system and the determination of Bitcoin market price. The second paper discusses three economic issues of Bitcoin system. First of all, how can we measure the fundamental value of Bitcoin system? Second, what is the investing strategy of the speculators in the Bitcoin trading market and how do the interactions among the different types of investors lead to the high volatility of Bitcoin market price? Third, what are the statistical characteristics of historical Bitcoin market price? Will the classical speculative features appear in the Bitcoin market price?

Moreover, the model of another essential mechanism of Bitcoin system-"Mining competition" is developed in the third paper. Each node who participates in the competition of Bitcoin block creation in Bitcoin network is defined as a Bitcoin miner. The miner who solves the puzzle first in each round of competition obtains the right to upload the candidate block, which contains the transaction information, to Bitcoin block chain. Additionally, the reward of each winner includes all the Bitcoin incorporated in the Bitcoin block and the transaction fee which is also denominated in Bitcoin. The previous economics literature on Bitcoin mining competition mainly focuses on how the miners collaborate to maximize the revenue through the organization of mining pools, while there is little analysis on the mining behaviour according to the Bitcoin protocol designed by Nakamoto. Therefore, in the third paper, a model is developed to study two issues. First, what is the optimal input of Bitcoin miners according to the Bitcoin protocol, given the exogenous market price of Bitcoin? Second, due to the capacity limitation of Bitcoin block, what is the optimal way to construct a candidate block for the Bitcoin miners?

The remainder of the dissertation is organized as follows. In Chapter 2, the model of two RMB exchange rates determination is displayed. The mechanism of Bitcoin market pricing is illustrated in Chapter 3 and a standard model of Bitcoin mining competition is developed in Chapter 4. Simulations and empirical analysis are performed in each chapter respectively. Finally, the summary and discussion are made in Chapter 5.

CHAPTER 2. ONE COUNTRY TWO SYSTEMS: FIXED AND FLEXIBLE EXCHANGE RATES

To be submitted

Abstract

For the first time, this study develops a theoretical model for the determination of two exchange rates of Renminbi (RMB) and their interactions, one in managed-floating onshore exchange market and the other one in free-floating offshore exchange market. Besides, the empirical analysis shows that the interactions between the two exchange rates and explanatory variables are time-varying. Particularly, the leading role was shifted from the onshore market to the offshore market after "811 Reform of RMB Exchange Rate". This research fills the gap in market micro-structure theory applied to RMB exchange rate.

Keywords: Onshore-Offshore RMB Exchange Rate, Market Micro-Structure, Intervention, Rational Expectation, Fundamentalist, Chartist, Vector Error Correction Model

2.1 Introduction

With the rapid growth of Chinese economy and trade surplus, the international use of the Renminbi (RMB) has increased significantly in recent years. The RMB ranks eighth in the most actively traded currencies worldwide, moving up one position from 2013, in the triennial survey of forex activity complied by Bank for International Settlements (BIS) ([1]). The establishment of offshore RMB foreign exchange system, in which Hong Kong market is the main trading center, is a principal symbol of RMB internationalization.

Although Hong Kong is geographically close to mainland of China, the RMB exchange rate in the offshore market, broadly referred as CNH, behaves differently from the RMB exchange rate in the onshore market, known as CNY. The offshore exchange rate is free-floating because capital flows freely in Hong Kong area. However, the dominant feature of onshore exchange rate is managed-floating with the intervention from the People's Bank of China (PBoC). Research on the two RMB exchange rates has been focused on the analysis of the time-varying relationship between the offshore and onshore exchange rates by using empirical methodologies ([2], [3]). So far, few studies have investigated on the determination of the two RMB exchange rates.

The purpose of this study is to explore the pricing mechanism of the two RMB exchange rates. Specifically, this paper aims at answering the following questions. First, given the onshore exchange rate, how do heterogeneous participants determine the offshore exchange rate? Second, in a sequential trading mechanism, how do commercial banks in the onshore inter-bank foreign exchange market maximize their values of trading and how does the PBoC manage the exchange rate, as a counter-party in the transaction? Third, what is the difference of statistical characteristics between the two RMB exchange rates in two regimes, before and after exchange rate reform in August 11^{th}, 2015?

To explore the above issues, this paper constructs a model to illustrate the dynamic linkages between the onshore and offshore RMB foreign exchange markets and the pricing mechanism in each market respectively. At each time point, the representative commercial banks in the onshore market observe the previous exchange rates and current effective exchange rate. In addition, they also generate expectations on the onshore exchange rate in the next transaction day. After incorporating the reaction function of commercial banks, the PBoC sets the onshore exchange rate through intervention. This official and final onshore exchange rate will be revealed immediately in the offshore market. Participants in the offshore market then refer to the onshore exchange rate to determine the exchange rate according to their individual forecasting rules. This is the first time to link the inter-bank foreign exchange market under the intervention of the central bank with an open free-floating foreign exchange market and to discuss their interactions, which is the major contribution of this paper.

A Vector Error-Correction Model (VECM) is applied to capture the dynamic correlations between the two RMB exchange rates and the explanatory variables over time in two regimes, before and after the threshold in August 11^{th}, 2015. These explanatory variables include the effective exchange rate index (EI), the inter-bank forward RMB exchange rate (YE), the offshore non-delivery forward RMB exchange rate (HE), and the foreign exchange reserves of the PBoC (FX). The estimated VECM paths overlay with the realized exchange rate series, indicating that the model captures most of the features of both onshore and offshore exchange rates. In addition, the results also suggest that the exchange rates with explanatory variables display different behaviour patterns before and after the exchange reform in 2015. After the reform, the influence of the onshore exchange rate on itself has been weakened, while the impact of offshore exchange rate has been strengthened, which is indirect evidence of RMB's internationalization. Moreover, the comparison between the paths of the two exchange rates simulated by the equilibrium equations of the theoretical model with the estimated path is performed. The results suggest that the effective exchange rate index is not a key factor in offshore exchange rate determination as expected, while the multi-lagged items of both exchange rates have impacts on the current level determination.

This paper comprises seven sections structured as follows. Introduction is provided in Section 2.1. In Section 2.2, literature on the intervention of central bank, market micro-structure theory and previous empirical analysis of the two RMB exchange rates are reviewed, especially with regard to the theoretical model of trading behaviour in the inter-bank and free-floating foreign exchange market. Section 2.3 provides a brief introduction to the mechanisms of RMB onshore and offshore foreign exchange markets and the PBoC's intervention. Section 2.4 is the core part of this study. A theoretical model in which offshore and onshore RMB exchange rates are determined is developed.

In addition, the mechanism of the PBoC's intervention is also modeled. In order to capture the statistical properties of the two RMB exchange rates, an empirical analysis is performed in Section 2.5. The calibration work is illustrated in Section 2.6. Section 2.7 provides a conclusion of the paper.

2.2 Literature Review

Using a framework of market micro-structure theory, the main objective of this study is to construct a dynamic linkage between the offshore free-floating foreign exchange market and the onshore managed-floating market, in which the PBoC performs intervention.

2.2.1 Intervention by the Central Bank

A series of recent studies ([4], [5], [6], [7], [8], [9], [10], [11], [12], [13], [14], [15], [16], [17], [18]) conduct research in currency market intervention by monetary authorities in various countries. Some literature ([4], [5], [6], [7] and [12]) provide the empirical evidence of "leaning-against-the-wind" operation. The more the market exchange rate deviates from the target value, the more likely monetary authorities perform intervention.

Besides this, another trigger of central bank's intervention is to reduce excessive volatility of exchange rate ([8], [10]). [17] study the development of onshore (Tokyo) and offshore (London & New York) foreign exchange markets to investigate the intervention performed by the Bank of Japan (BoJ) from 1991 to 2004. They claim that the excess exchange rate volatility is the incitement of intervention performed by the central bank. Additionally, the evidence of "leaning-against-the-wind" during Yen depreciation period between 2003 and 2004 is found in their research.

Generally, investors adjust their portfolio allocation according to the expected return rates in different countries. The effect of sterilized intervention from monetary authorities in the currency market may not be completely off-set when the foreign asset is not a perfect substitute for the domestic asset. The representative literature that demonstrates this intervention of "Portfolio Balance Channel" includes [19], [20], [21], [22], [23], [24] and [25].

2.2.2 Market Micro-Structure

In the "Portfolio Balance Channel" model, the exchange rate will only be adjusted due to the unexpected changes in fundamental macro-economic variables (inflation, output growth, interest rates, etc) ("News"). Some studies ([26], [27], [28], [29] and [30]) reveal that the model failed in determining exchange rate empirically. Most of the exchange rate adjustment happens without fundamental macro-economic news being released. In other words, the realized exchange rate seems to be disconnected from its fundamentals.

Market structure theory ([31], [32], [33], [34] and [35]) provides an alternative approach to analyze exchange rate determination. Private information, transaction mechanisms and heterogeneity of agents are determinants of the exchange rates. In the framework of [35], the market agents, which are classified as fundamentalists and chartists, make predictions on future exchange rates according to their individual rules. On one side, fundamentalists perform their predictions by comparing between spot and fundamental exchange rates. In other words, they believe the deviated market rates will converge to the fundamental rate eventually. On the other side, chartists deduce the change of exchange rate by calculating the moving average of the previous rates. Compared to the forecasting rule of fundamentalists, the fundamental exchange rate is merely referred indirectly by chartists. In this sense, chartists may be considered as "noise traders". Their charting rules, to some extent, reflect a herd effect.

Under the framework of market structure, a series of studies ([22], [36], [23] and [37]) concentrate on how the intervention by monetary authorities affects the spot exchange rate. Given the assumption that not only the domestic and foreign assets are not perfectly substituted, but also the central bank may has "dominant" economic fundamental information and is willing to release these information, the agents in foreign exchange market will modify their expectations on the exchange rate. Moreover, [38] argues that the spot exchange rate can be affected as well even if the central bank does not disclose all its "dominant" information. Since central bank is concerned about its profits and losses, then the agents in foreign exchange market are convinced that the intervention provides creditable information about monetary policy in the future.

[39] proposes a model in which monetary authorities manage the floating exchange rate regularly. The effective exchange rate is independent of any operations by monetary authority, and central bank's intervention is considered as sterilized. The primary purposes of central bank's intervention are to prevent the spot exchange rate from deviating from a specific target level and to stabilize the fluctuation of exchange rate. In addition, the central bank, like other market agents, is short-sighted at each point in time. Thus, the central bank will not return to the market after each operation is completed. Moreover, the value function of central bank's intervention reflects the contradictory motivations of monetary policy and wealth preservation (or speculation).

Concentrating on the intervention of central banks in emerging economies, [40] built a model of the transaction process in the inter-bank foreign exchange market. Under this trading mechanism, the buyers and sellers submit their orders of foreign exchange according to a specific price which is usually the previous closing price. The value function of buyers and sellers depends on the exogenous economic fundamentals and the transaction volume, which is a function of the exchange rate level. The market price is always adjusted in the same direction as the excess demand and the equilibrium is formed when all orders are executed. This study is critical to the research because the onshore RMB exchange market is a typical inter-bank foreign exchange market. In the designed sequential transaction of the onshore exchange market, given all public information, the commercial banks submit their orders to the PBoC which has strong market power. Then the PBoC fills the orders and then onshore exchange rate is determined.

2.2.3 Onshore and Offshore Exchange Rate

The literature investigating interactions between onshore and offshore exchange rates ([41], [42], [43], [44], [45] and [46]) mainly focus on measuring time-varying difference and impact from economic variables. Among these literature, [44] used empirical methodologies to investigate the causality relationship between the RMB onshore spot and forward markets, and that between RMB offshore spot and forward markets. Additionally, [45] studied the relationship between RMB onshore and offshore exchange rates using VECM. The empirical results suggest that the information-guided dynamic relationship between both the onshore and offshore spot exchange rates and forwards are significant. Moreover, [46] studied the interaction between these two RMB forward markets and found that most of time the impact of onshore on offshore is greater than that in the opposite direction.

2.2.4 Summary

In summary, traditional macro-economic exchange rate determination theory is insufficient to explain the short-term behaviour of market exchange rates. The long-term deviation between the market exchange rates and the fundamental economy has been confirmed in a series of studies. Market micro-structure theory attempts to provide alternative approaches to explain the determination of the market exchange rate. Under this framework, the aim of central bank's daily sterilized intervention is to reduce volatility and to narrow the gap between market exchange rate and the target level. In our research, the structure of the onshore inter-bank foreign exchange market, which has been neglected in most of the previous literature, has essential practical value to explain the determination of onshore RMB exchange rate. This market is not perfectly competitive. Instead, it consists of agents with market power playing in a sequential trading mechanism. Meanwhile, the offshore RMB foreign exchange market is a standard free-floating market that can be characterized by models provided in previous literature. In Section 2.3, more details of the RMB exchange rate mechanism will be developed.

2.3 Description of RMB Exchange Rate Mechanism

The following subsections introduce general information about the free-floating CNH and the managed-floating CNY with foreign exchange intervention of the PBoC, respectively.

2.3.1 Onshore RMB Exchange Rate System

The mainland foreign exchange market was established in 1994 which represents the abolition of the official exchange rate. Then, China has been adopting a very narrow range managed-floating exchange rate system until 2005. During this period, the exchange rate between RMB and U.S. Dollar was fixed around 8.28. On July 21^{st}, 2005, the PBoC initiated a reform of RMB exchange rate system, announcing a 2.1% appreciation of RMB against U.S. Dollar, and a program to transfer the actual pegging exchange rate system to a managed-floating system. In addition, a wider range of exchange rate fluctuation was allowed, with (foreign exchange) market demand-supply relationship having a more significant impact on exchange rate determination. Therefore, the original official RMB exchange rate which was previously determined completely by the PBoC was changed to being determined by many factors.

Moreover, on August 11^{th}, 2015, the PBoC announced that it would improve the quotation mechanism for RMB against U.S. Dollar ("8·11 Reform of RMB Exchange Rate"). Before the opening of each day's onshore inter-bank foreign exchange market, each market agent will provide a quotation price to the China Foreign Exchange Trading Center (CFETC), with reference to the supply-demand relationship, the recent movement of major exchange rates and the previous closing prices in both onshore and offshore markets. The PBoC will match the quotations considering the exchange rate trend of "one basket of currency". The most significant change in "8·11 Reform of RMB Exchange Rate" is that more information about the previous transactions, in both onshore and offshore markets, is incorporated in the determination of CNY. The influence of free-floating CNH was increased, reflecting the process of marketization of RMB exchange rate pricing.

In general, under a managed-floating exchange rate system, the primary purpose of the central bank's foreign exchange intervention is to stabilize the spot exchange rate within a limited range. Compared with the intervention of monetary authorities in other countries, the PBoC is a heavy interventionist, whose intervention is an essential determinant of the daily RMB exchange rate. The PBoC rarely intervenes in the foreign exchange rate by adjusting domestic interest rates or the reserve ratio of commercial banks. Instead, it intervenes in the foreign exchange market in three main ways: OMO, setting the Central Parity of exchange rate, and oral intervention.

OMO is the most common way of intervention. The PBoC intervenes in the exchange rate through purchasing or selling foreign assets (currency) directly. In the case of the purchasing operations, the PBoC purchases foreign currency using its central bank bills; for the selling operations, the PBoC sells the foreign exchange reserves. This type of intervention is mainly operated in the inter-bank foreign exchange market, which is a unified foreign exchange wholesale market in CFETC. The participants in inter-bank foreign exchange market include commercial banks authorized for foreign exchange transactions, other financial agents certificated for foreign exchange business, and the PBoC which performs intervention.

The second way the PBoC intervenes is by setting a central parity of the RMB exchange rate, which restricts the daily fluctuation into a specific range. Finally, oral intervention, which includes policy statement and warning, can be considered as the third type of intervention. Due to the fact that the effectiveness of these two types of intervention are difficult to be determined, the intervention by the OMO is always the primary and widely adopted choice.

2.3.2 Offshore RMB Exchange Rate System

Similar to market variables such as the spot exchange rate, the forward exchange rate and the central-parity price of the onshore RMB market, the price system of the offshore RMB market includes the corresponding spot exchange rate, forward exchange rate, and central-parity price. On June 27^{th}, 2011, the Hong Kong Treasury Market Association (HKTMA) launched the official RMB-U.S. Dollar spot exchange rate. Fifteen banks in Hong Kong designated by HKTMA provide quotation services. After getting rid of the three highest and lowest prices, the average price is set as the central-parity price. This central parity price is a benchmark for the offshore RMB exchange rate and the development of offshore RMB exchange derivatives.

Although Hong Kong is geographically close to mainland China, the offshore RMB exchange market in Hong Kong is different from the corresponding onshore RMB exchange market in many aspects. There exists strong capital control in mainland of China, whereas capital flows in Hong Kong face much less limitation. Effective segregation caused by capital control makes it possible for the two exchange rates to coexist for one currency. CNY is determined in the onshore inter-bank foreign exchange market, while the free-floating CNH is determined by the offshore market participants. In the absence of price floating ranges and capital control, CNH can better reflect pure market factors and economic situations for the RMB exchange rate.

Moreover, the persistent deviation between the historical offshore and onshore exchange rate justifies the effective capital control and limited arbitrage opportunities. In general, arbitrage is performed through the trade settlement and capital flows. However, both of them are strictly restricted by the onshore monetary authorities. Specifically, although the enterprises engaged in import and export can choose between the onshore and offshore exchange rates to settle their trade, the authorities require the settlement to be accompanied by the real production and trading. Specifically, the enterprises engaged in import and export must set up the special accounts for their positions of foreign exchange. The authorities censor these accounts and conduct the authenticity audit of the trade, based on the three principles: Know your customer (KYC), know your customer's business (KYB), and customer due diligence (CDD). Therefore, it is guaranteed that the enterprises can not benefit from fraud trading.

Besides, while both domestic and foreign investors can invest in RMB bonds and deposits and exchange RMB with U.S. Dollar freely in the offshore market, in the onshore market, however, the domestic investors can only purchase no more than $50,000 annually, and the foreign investors are not permitted to invest in RMB and related financial products. Overall, it is ensured that the arbitrage between two RMB exchange rates is narrow and can not be easily implemented.

2.4 Theoretical Model

In this section the general model is presented, which illustrates the dynamic linkages between the onshore and offshore RMB foreign exchange markets. The pricing mechanism in each individual market is mainly based on [40], [39] and [35].

The PBoC, mainland commercial banks and authorized financial institutions which are designated for foreign exchange transactions are the main participants in the onshore RMB exchange market, *i.e.*, inter-bank foreign exchange market. On the other hand, individual financial agents are the main market players in the offshore exchange market. The dynamic linkage of onshore exchange market and offshore exchange market can be formulated in the following way. Let the time horizon for all agents be infinite. In each period t, a representative commercial bank in the onshore market observes the offshore exchange rate of last period $\tilde{e}_{t-1}$ and the current effective exchange rate $\bar{e}_t$

which is exogenously defined as the ratio of the price level abroad and the domestic price level. In addition, the representative commercial bank also forms the expectation of next onshore market exchange rate $E_t(e_{t+1})$ as well.

The central bank (the PBoC) sets the onshore exchange rate e_t through foreign exchange intervention after taking into account the reaction function of the commercial banks. This official and final onshore exchange rate of period t will be known in the offshore market instantaneously. Then participants in the offshore market determine the exchange rate $\tilde{e}_t$ through their individual pricing mechanism with reference to e_t. It must be noted that the difference between central parity and the onshore exchange rate is ignored in this model.

In following subsections, the details of dynamic pricing in the two markets will be introduced. Since the offshore market exchange rate depends on the onshore market rate, and to the extent that the offshore rate affects the foreign exchange demand of onshore commercial banks during the next period, then the central bank takes the offshore equilibrium function into account before intervening in the onshore markets as well.

2.4.1 Determination of Offshore RMB Exchange Rate

Unlike other economic studies analyzing how macroeconomic variables determine effective exchange rates, this research focuses on how the foreign exchange market rate is determined by the behaviour of market participants. The free-floating offshore RMB foreign exchange market is populated by individual investment agents. The Hong Kong Monetary Authorization (HKMA) is the market monitor and does not perform any intervention. [35] stated that market participants understand they have cognitive limitations in processing and evaluating information. They cannot deal with all complex issues. Therefore, market participants usually follow simple and straightforward rules to guide their trading behaviour in the market.

[35] categorized participants in foreign exchange market as fundamentalists and chartists. Fundamentalists make their foreign exchange investment base on the comparison of the offshore exchange rate and the effective exchange rate. Their belief is that the spot exchange rate should fluctuate with the real effective exchange rate. Hence, in our model, the fundamentalists' expectation can be written as:

$$E_t^F(\tilde{e}_{t+1}) = \tilde{e}_t + \lambda(\bar{e}_t - \tilde{e}_t) + \mu(e_t - \tilde{e}_t) \tag{2.1}$$

where λ, μ >0. When the current offshore exchange rate $\tilde{e}_t$ is below the effective exchange rate $\bar{e}_t$ or below onshore market exchange rate e_t, fundamentalists forecast that the offshore exchange rate will rise in the future. In the opposite case where the reverse case is current offshore exchange rate exceeds the effective exchange rate and the onshore market exchange rate, the fundamentalists believe the offshore exchange rate will decline.

In contrast, the chartists form their expectations through following the direction of the historical exchange rate trend. For simplicity, chartists are assumed to be short-sighted and only concerned about price change between the current and the last period.

$$E_t^C(\tilde{e}_{t+1}) = \tilde{e}_t + \alpha(\tilde{e}_t - \tilde{e}_{t-1}) \tag{2.2}$$

where α >0. Chartists predict the exchange rate will continue to rise (decline) when the exchange rate has increased (declined).

Suppose the excess demand function for foreign currency of these two types of participants are:

$$x_t^F = \pi^F(\tilde{e}_t - E_t^F(\tilde{e}_{t+1})) + \omega^F = \pi^F(\lambda(\tilde{e}_t - \bar{e}_t) + \mu(\tilde{e}_t - e_t)) + \omega^F; \tag{2.3}$$

$$x_t^C = \pi^C(\tilde{e}_t - E_t^C(\tilde{e}_{t+1})) + \omega^C = \pi^C(\alpha(\tilde{e}_{t-1} - \tilde{e}_t)) + \omega^C \tag{2.4}$$

The exchange rate is defined as the value of U.S. Dollar equivalent to one unit of RMB. Therefore, according to equations (2.3) and (2.4), the participants purchase the foreign currency at current state if they forecast exchange rate decreasing in the future, *i.e.*, one RMB will exchange to fewer U.S. Dollars.

Here π^F and π^C are positive parameters which measure the sensitivity of demand to the deviation. ω^F and ω^C are constant parameters. In addition, compared with the model in [35], the impact from second-order moment (volatility) on pricing is ignored here and normalized as one.

Moreover, a stable relationship is assumed existing between the equilibrium supply of the U.S. Dollar and the effective exchange rate:

$$\bar{S}_t = \eta \bar{e}_t \tag{2.5}$$

where $\bar{S}_t$ is a equilibrium supply of U.S. Dollar. There is a one-to-one relationship between the effective exchange rate and the equilibrium supply of U.S. Dollar. The extent to which the effective exchange rate changes affect the equilibrium supply of net U.S. Dollar is measured by the parameter η.

The dynamic mechanism of U.S. Dollar supply can be expressed as follows:

$$S_t - \bar{S}_t = \psi(S_t - \bar{S}_{t-1}) + (1-\psi)\eta(\tilde{e}_t - \bar{e}_t) \tag{2.6}$$

Equation (2.6) shows that the supply of U.S. Dollar is higher than its equilibrium level when the market exchange rate is higher than the effective exchange rate. According to the definition of exchange rate which is the value of U.S. dollar equalling to one unit of RMB, when the market exchange rate is higher than its effective value, the U.S. Dollar depreciates against the RMB. This will lead to more U.S. exports and fewer U.S. imports and thus improve the current account. However, the latter improvement implies that supply of U.S. Dollar to domestic agents rises to a level which is higher than its equilibrium supply. In general, this dynamic process of adjustment is gradual. The parameter ψ determines the speed of adjustment rate of the U.S. Dollar supply. $\psi = 0$ means that the adjustment is instantaneous; $\psi > 0$ implies a slower speed of adjustment. Since this study focuses on the dynamic relationship between offshore and onshore exchange rates, then the supply of U.S. Dollar is assumed to adjust simply to the change of exchange rate without any delay.

Here, the market supply of U.S. Dollar is determined by the exogenous net current account and by the purchasing or selling operation of Federal Reserve.

$$S_t - \bar{S}_t = \eta(\tilde{e}_t - \bar{e}_t) \tag{2.7}$$

Finally, the market clearing condition implies that aggregate demand of both fundamentalists and chartists equals to the foreign exchange supply.

$$S_t = x_t^F + x_t^C \tag{2.8}$$

By combining equations (2.3) - (2.8), the formula of offshore exchange rate is obtained:

$$\tilde{e}_t = \rho_0 + \rho_1 e_t + \rho_2 \bar{e}_t + \rho_3 \tilde{e}_{t-1} \tag{2.9}$$

where $\rho_0 = \frac{\omega^F + \omega^C}{\eta + \pi^C \alpha - \pi^F \lambda - \pi^F \mu}$; $\rho_1 = \frac{-\pi^F \mu}{\eta + \pi^C \alpha - \pi^F \lambda - \pi^F \mu}$; $\rho_2 = \frac{-\pi^F \lambda}{\eta + \pi^C \alpha - \pi^F \lambda - \pi^F \mu}$; $\rho_3 = \frac{\pi^C \alpha}{\eta + \pi^C \alpha - \pi^F \lambda - \pi^F \mu}$.

In above equation (2.9), the offshore exchange rate $\tilde{e}_t$ is determined by the onshore exchange rate e_t, the effective exchange rate $\bar{e}_t$, and its own lagged value $\tilde{e}_{t-1}$.

2.4.2 Determination of the Onshore RMB Exchange Rate

In this section, we will establish a spot onshore foreign exchange market trading model. The onshore RMB foreign exchange market is also known as the inter-bank foreign exchange market. Its structure can be modelled as a Stackelberg leader-follower model, in which the central bank leads by intervening in the foreign exchange market with a particular spot exchange rate in mind, because the central bank understands how the market participants will respond to the spot exchange rate. For simplicity, it is assumed that all participants are perfectly competitive and homogenous. While a representative competitive participant chooses the optimal transaction volume as the best reaction of the spot exchange rate, the PBoC sets the exchange rate by maximizing its valuation of intervention in onshore exchange market.

2.4.2.1 The Problem of a Commercial Bank

[40] builds a game-theoretic model to describe the participants in the inter-bank exchange market. For simplicity, we assume perfect competition in the banking sector and normalize the mass of the banking sector to unity. The objective function of a representative participant in the market can be written as:

$$\operatorname*{Max}_{x} V(x) - e \cdot x \tag{2.10}$$

where x is the transaction quantity. Positive (negative) x implies purchasing (selling) foreign currency. As in the previous section, e is the spot exchange rate which is denoted as the value of U.S Dollars per RMB. Hence, $e(\cdot)$ is determined by the aggregate transactions in the market. $V(x)$ is its value function of transaction volume.

The specific form of a commercial bank's value function is as follows:

$$V(x_t) = \gamma(e_t - \bar{e}_t)x_t + \chi(e_t - \tilde{e}_{t-1})x_t + (1 - \gamma - \chi)(E_t(e_{t+1}) - e_t)x_t - \frac{\delta x_t^2}{2} \tag{2.11}$$

where γ, χ, δ are positive parameters, in which γ, χ are restricted to be in the interval (0,1) satisfying $\gamma + \chi > 1$.

Equation (2.11) reflects the following forces. Suppose that a participant in the inter-bank foreign exchange market at the beginning of period, observes the spot exchange rate e_t is lower (higher) than the level of effective rate $\bar{e}_t$, which means that U.S. Dollar appreciates (depreciates) against RMB, it would like to sell (purchase) the foreign currency.

Similarly, the offshore exchange rate also provides an evaluation reference. If spot exchange rate is lower (higher) than the level of previous offshore market rate $\tilde{e}_{t-1}$, the participant also sells (purchases) the foreign currency. Additionally, market participant sells (purchases) the exchange as well if its exchange rate forecasting $E_t\left(e_{t+1}\right)$ is lower (higher) than the level of onshore spot rate.

Following the commercial bank's objective function (2.10), its optimal transaction volume for a given onshore spot exchange rate is derived from the first order condition:

$$V'(x_t) = e_t \tag{2.12}$$

Given the specific form of $V\left(x\right)$ in equation (2.11), we have:

$$x_t = \kappa_1 \bar{e}_t + \kappa_2 \tilde{e}_{t-1} + \kappa_3 E_t(e_{t+1}) + \kappa_4 e_t \tag{2.13}$$

where $\kappa_1 = -\frac{\gamma}{\delta}$, $\kappa_2 = -\frac{\chi}{\delta}$, $\kappa_3 = \frac{1-\gamma-\chi}{\delta}$, $\kappa_4 = \frac{2(\gamma+\chi-1)}{\delta}$

Equation (2.13) proposes that the foreign exchange reaction function of onshore commercial bank which depends on onshore spot exchange rate e_t, the effective exchange rate $\bar{e}_t$, previous offshore exchange rate $\tilde{e}_{t-1}$ and expectation $E_t\left(e_{t+1}\right)$. The coefficients illustrate that for the participants in inter-bank foreign exchange market, demand of foreign exchange is positive to the onshore spot exchange rate, but negative to the effective exchange rate, previous offshore exchange rate and the expectation of onshore exchange rate in the next period.

2.4.2.2 Central Bank

Central bank intervenes in the foreign exchange market persistently for several reasons. For example, to manage free-floating exchange rate, to be in compliance with international currency agreements; to serve the domestic macroeconomic agenda, or due to the domestic political pressure. While usually central bank's intra-day market operations look speculative, in particular, its motivation of policy and wealth preservation (or speculation) may conflict with each other. For example, if a representative participant in the inter-bank foreign exchange market forecasts that onshore spot exchange rate is higher than the equilibrium level, that is, U.S. Dollar depreciates against RMB, then it will send a request for purchasing foreign exchange to the central bank. After receiving this request, the central bank (the PBoC) will sell foreign exchange or buy the local currency (RMB) in the market. However, the sudden appreciation of domestic currency may cause deflationary pressure. To mitigate this effect, the central bank may purchase some foreign exchange (U.S. Dollar) in the market. Since the effective exchange rate is public, then the central bank will not maximize profits (Only to sell U.S Dollars will) if its actions are effective. This will lead to a reduction of value of wealth in expected future.

[39] summarizes these two effects. In this paper, for simplicity, we restricted central bank's operations in purchasing or selling foreign exchange reserve (U.S. Dollar), by postulating that the central bank's intervention loss function $L(\cdot)$ as follows:

$$L(e_t) = (e_t - e_{t-1})^2 - \theta E_t(e_{t+1})(X_t + \overline{FX}_{t-1}). \tag{2.14}$$

In above equation, X_t represents the amount of current foreign exchange operates (purchasing or selling) by the central bank, and FX_{t-1} stands for the accumulative foreign exchange reserve before intervention, which is set as an exogenous variable. The first component in this equation measures the policy motives of the central bank to smooth fluctuations of exchange rate. The second component contains the motivation of wealth preservation. The intervention is not cost-less when the operation of the central bank is not profitable. It reflects the possible wealth change of the central bank in the next period due to operations.

Parameter θ balances the potential correlation between policy and speculation, which can be interpreted as a measure of the central bank's commitment to stabilizing the exchange rate. $\theta = 0$ indicates that the central bank, performing like a pure price manipulator, does not concern about the cost of intervention. $\theta > 0$ implies that the wealth preservation is matter to the central bank, the more so the larger θ. Since the transaction of currency becomes profitable, the loss function is sensitive to the changes in its wealth.

The amount of foreign exchange operations of central bank should be offset by the excess demand of participants in the inter-bank foreign exchange market, so the clearing condition of the onshore foreign exchange market is :

$$X_t + x_t(e_t) = 0 \tag{2.15}$$

By substituting (2.15) back into (2.14) and take the first order condition of (2.14) with respect of e_t, the equilibrium demand of exchange, which depends on exchange rate, must satisfy:

$$2e_t - 2e_{t-1} + \theta \frac{\partial E_t\left(e_{t+1}\right)}{\partial e_t} x_t - \theta \frac{\partial E_t\left(e_{t+1}\right)}{\partial e_t} \overline{FX}_{t-1} + \theta \frac{\partial x_t}{\partial e_t} E_t(e_{t+1}) = 0 \tag{2.16}$$

Equation (2.16) describes the dynamic path of onshore exchange rate e_t. We will solve out the explicit form of e_t in the next section.

2.4.3 Equilibrium

In order to solve e_t in equation (2.16) explicitly, e_t is conjectured to behave according to the following process:

$$e_t = \tau + \xi e_{t-1} + \varphi \bar{e}_t + \Omega \overline{FX}_{t-1} + \nu \tilde{e}_{t-1} \tag{2.17}$$

Through observing the specific form of equation (2.9) and (2.16), we provide the linear conjecture form of e_t as equation (2.17). The previous onshore exchange rate e_{t-1}, current effective exchange rate $\bar{e}_t$ and the previous offshore exchange rate level $\tilde{e}_{t-1}$ determine the current onshore exchange rate level together. Here τ, ξ, φ, Ω and ν are parameters to be determined. τ is the intercept item. Therefore, expectation of onshore exchange rate $E_t\left(e_{t+1}\right)$ is conjectured rationally as:

$$E_t(e_{t+1}) = E_t\{\tau + \xi e_t + \varphi \bar{e}_{t+1} + \Omega \overline{FX}_t + \nu \tilde{e}_t\} \tag{2.18}$$

At period t, only the effective exchange rate in the next period is exogenous. Hence,

$$E_t(e_{t+1}) = \tau + \xi e_t + \varphi E_t(\bar{e}_{t+1}) + \Omega \overline{FX}_t + \nu \tilde{e}_t \tag{2.19}$$

Empirical evidence illustrates that the series of market indices (prices) tend to have small, positive average. Hence, exogenous $\bar{e}_t$ is supposed to follow the pattern of random walk with drift :

$$\bar{e}_t = \Phi + \bar{e}_{t-1} + a_t \tag{2.20}$$

where a_t is random disturbance item.$a_t \sim i.i.d.N(\Phi, \sigma_\varepsilon)$. The constant term Φ represents the trend of the effective exchange rate sequence, which is often referred as drift of the sequence.

This setting makes the effective exchange rate consist of cumulative time trend $t\Phi$ and a pure random walk process with $Var(\sum_{i=1}^{t} \varepsilon_i) = t\sigma_\varepsilon^2$, σ_ε^2 is variance of ε_t. Additionally, the conditional standard deviation of $\bar{e}_t$ is $\sqrt{t}\sigma_\varepsilon$, of which the growth rate is lower than the conditional expectation of $\bar{e}_t$. Therefore, if we map series $\bar{e}_t$ over time t, a dynamic trend with slopes evolves. A positive (negative) slope indicates that the exchange rate rise (decrease) as t rises. By substituting equation (2.20) into (2.19), we have:

$$E_t(e_{t+1}) = \tau + \varphi\Phi + \xi e_t + \Phi\bar{e}_t + \Omega\overline{FX}_t + \nu\tilde{e}_t \tag{2.21}$$

In addition, the amount of central bank's foreign exchange reserves should satisfy the relation:

$$\overline{FX}_t = X_t + \overline{FX}_{t-1} = -x_t + \overline{FX}_{t-1} \tag{2.22}$$

Equation (2.22) shows that the cumulative foreign exchange reserve at the end of the current period equals to its level of last period with that used in the intervention. By combining equations (2.21) with (2.9) and (2.13), then expectation of the onshore exchange rate can be written as:

$$E_t(e_{t+1}) = \frac{\tau + \varphi\Phi + \rho_0}{1+\kappa_3\Omega} + \frac{\xi + \rho_1\nu - \kappa_4\Omega}{1+\kappa_3\Omega}e_t + \frac{\varphi + \rho_2\nu - \kappa_1\Omega}{1+\kappa_3\Omega}\bar{e}_t + \frac{\rho_3\nu - \kappa_2\Omega}{1+\kappa_3\Omega}\tilde{e}_{t-1} + \frac{\Omega}{1+\kappa_3\Omega}\overline{FX}_{t-1} \tag{2.23}$$

From this conjecture form of expectation, expectation's derivative with respect to the spot exchange rate is:

$$\frac{\partial E_t(e_{t+1})}{\partial e_t} = \frac{\xi + \rho_1\nu - \kappa_4\Omega}{1 + \kappa_3\Omega} \tag{2.24}$$

Using this result (2.24), take the first order condition of x_t regard to e_t in equation (2.13), then we have:

$$\frac{\partial x_t}{\partial e_t} = \frac{\kappa_3\xi + \kappa_3\rho_1\nu - \kappa_3\kappa_4\Omega}{1 + \kappa_3\Omega} + \kappa_4 \tag{2.25}$$

So far, we have collected all the elements that describe the dynamic path of onshore exchange rates. By substituting (2.13), (2.23), (2.24) and (2.25) back into (2.16), then we have relation among these variables as follow:

$$\begin{aligned}
&\{2 + \theta[\frac{\kappa_4(\xi + \rho_1\nu - \kappa_4\Omega)}{1 + \kappa_3\Omega}] + \theta\{\frac{[\kappa_3(2\xi + 2\rho_1\nu - \kappa_4\Omega) + \kappa_4](\xi + \rho_1\nu - \kappa_4\Omega)}{(1 + \kappa_3\Omega)^2}\}\}e_t - 2e_{t-1} + \\
&\{\theta[\frac{\kappa_1(\xi + \rho_1\nu - \kappa_4\Omega)}{1 + \kappa_3\Omega}] + \theta\{\frac{[\kappa_3(2\xi + 2\rho_1\nu - \kappa_4\Omega) + \kappa_4](\varphi + \rho_2\nu - \kappa_1\Omega)}{(1 + \kappa_3\Omega)^2}\}\}\bar{e}_t + \\
&\{\theta[\frac{\kappa_2(\xi + \rho_1\nu - \kappa_4\Omega)}{1 + \kappa_3\Omega}] + \theta\{\frac{[\kappa_3(2\xi + 2\rho_1\nu - \kappa_4\Omega) + \kappa_4](\rho_3\nu - \kappa_2\Omega)}{(1 + \kappa_3\Omega)^2}\}\}\tilde{e}_{t-1} + \\
&\{\theta[\frac{\xi + \rho_1\nu - \kappa_4\Omega}{1 + \kappa_3\Omega}] - \theta\{\frac{[\kappa_3(2\xi + 2\rho_1\nu - \kappa_4\Omega) + \kappa_4]\Omega}{(1 + \kappa_3\Omega)^2}\}\}\overline{FX}_{t-1} + \\
&\theta\{\frac{[\kappa_3(2\xi + 2\rho_1\nu - \kappa_4\Omega) + \kappa_4](\tau + \varphi\Phi + \rho_0)}{(1 + \kappa_3\Omega)^2}\} \\
&= 0
\end{aligned} \tag{2.26}$$

If our prediction of the onshore exchange rate is rational, then e_t in equation (2.26) must have the same form as that in equation (2.17). Thus, parameters τ, ξ, φ, Ω, ν satisfy:

$$\tau = \frac{-\theta\{\frac{[\kappa_3(2\xi + 2\rho_1\nu - \kappa_4\Omega) + \kappa_4](\tau + \varphi\Phi + \rho_0)}{(1 + \kappa_3\Omega)^2}\}}{\{2 + \theta[\frac{\kappa_4(\xi + \rho_1\nu - \kappa_4\Omega)}{1 + \kappa_3\Omega}] + \theta\{\frac{[\kappa_3(2\xi + 2\rho_1\nu - \kappa_4\Omega) + \kappa_4](\xi + \rho_1\nu - \kappa_4\Omega)}{(1 + \kappa_3\Omega)^2}\}\}} \tag{2.27}$$

$$\xi = \frac{2}{\{2 + \theta[\frac{\kappa_4(\xi+\rho_1\nu-\kappa_4\Omega)}{1+\kappa_3\Omega}] + \theta\{\frac{[\kappa_3(2\xi+2\rho_1\nu-\kappa_4\Omega)+\kappa_4](\xi+\rho_1\nu-\kappa_4\Omega)}{(1+\kappa_3\Omega)^2}\}\}} \tag{2.28}$$

$$\varphi = \frac{-\{\theta[\frac{\kappa_1(\xi+\rho_1\nu-\kappa_4\Omega)}{1+\kappa_3\Omega}] + \theta\{\frac{[\kappa_3(2\xi+2\rho_1\nu-\kappa_4\Omega)+\kappa_4](\varphi+\rho_2\nu-\kappa_1\Omega)}{(1+\kappa_3\Omega)^2}\}\}}{\{2 + \theta[\frac{\kappa_4(\xi+\rho_1\nu-\kappa_4\Omega)}{1+\kappa_3\Omega}] + \theta\{\frac{[\kappa_3(2\xi+2\rho_1\nu-\kappa_4\Omega)+\kappa_4](\xi+\rho_1\nu-\kappa_4\Omega)}{(1+\kappa_3\Omega)^2}\}\}} \tag{2.29}$$

$$\Omega = \frac{-\{\theta[\frac{\kappa_2(\xi+\rho_1\nu-\kappa_4\Omega)}{1+\kappa_3\Omega}] + \theta\{\frac{[\kappa_3(2\xi+2\rho_1\nu-\kappa_4\Omega)+\kappa_4](\rho_3\nu-\kappa_2\Omega)}{(1+\kappa_3\Omega)^2}\}\}}{\{2 + \theta[\frac{\kappa_4(\xi+\rho_1\nu-\kappa_4\Omega)}{1+\kappa_3\Omega}] + \theta\{\frac{[\kappa_3(2\xi+2\rho_1\nu-\kappa_4\Omega)+\kappa_4](\xi+\rho_1\nu-\kappa_4\Omega)}{(1+\kappa_3\Omega)^2}\}\}} \tag{2.30}$$

$$\nu = \frac{\{\theta[\frac{\xi+\rho_1\nu-\kappa_4\Omega}{1+\kappa_3\Omega}] - \theta\{\frac{[\kappa_3(2\xi+2\rho_1\nu-\kappa_4\Omega)+\kappa_4]\Omega}{(1+\kappa_3\Omega)^2}\}\}}{\{2 + \theta[\frac{\kappa_4(\xi+\rho_1\nu-\kappa_4\Omega)}{1+\kappa_3\Omega}] + \theta\{\frac{[\kappa_3(2\xi+2\rho_1\nu-\kappa_4\Omega)+\kappa_4](\xi+\rho_1\nu-\kappa_4\Omega)}{(1+\kappa_3\Omega)^2}\}\}} \tag{2.31}$$

After re-arrangement, we have solved the parameters τ, ξ, φ, Ω, ν explicitly in the expression of equilibrium e_t, the below five equations build the linkage among them:

$$\begin{aligned}
&2\theta\xi(\varphi\Phi + \rho_0)\kappa_3 + \varphi(\theta\Phi\kappa_4 - \theta\Phi\Omega\kappa_3\kappa_4 + 2\theta\nu\Phi\kappa_3\rho_1) + \\
&\tau(2 + 2\theta\xi^2\kappa_3 + \theta\kappa_4 + 2\Omega^2\kappa_3^2 + \theta\kappa_4 + 2\theta\nu^2\kappa_3\rho_1^2 + \\
&\xi(2\theta\kappa_3 + 2\theta\kappa_4 - 2\theta\Omega\kappa_3\kappa_4 + 4\theta\nu\kappa_3\rho_1) + \\
&\nu(2\theta\kappa_3\rho_1 + 2\theta\kappa_4\rho_1) + \\
&\Omega(4\kappa_3 - \theta\kappa_3\kappa_4 - 2\theta\kappa_4^2 - 2\theta\nu\kappa_3\kappa_4\rho_1)) = 0
\end{aligned} \tag{2.32}$$

$$\begin{aligned}
&-2 + 2\theta\xi^2\kappa_3 - 4\Omega\kappa_3 - 2\Omega^2\kappa_3^2 + \\
&\xi^2(2\theta\kappa_4 - 2\theta\Omega\kappa_3\kappa_4 + 4\theta\nu\kappa_3\rho_1) + \\
&\xi(2 + 2\Omega^2\kappa_3^2 + 2\theta\nu\kappa_4\rho_1 + 2\theta\nu^2\kappa_3\rho_1^2) + \\
&\xi(\Omega(2(2\kappa_3 - \theta\kappa_4^2) - 2\theta\nu\kappa_3\kappa_4\rho_1)) = 0
\end{aligned} \tag{2.33}$$

$$
\begin{aligned}
&\theta\kappa_1(\xi - 2\Omega\kappa_4 + \nu\rho_1 - \Omega\kappa_3(\xi + \nu\rho_1))+\\
&\varphi(2 + 2\Omega^2\kappa_3^2 + \theta\kappa_4(1 + 2\xi - 2\Omega\kappa_4 + 2\nu\rho_1)+\\
&\kappa_3(-\theta\Omega\kappa_4(1 + 2\xi + 2\nu\rho_1) + 2(\theta\xi(1 + \xi)+\\
&2\Omega + \theta\nu\rho_1(1 + 2\xi + \nu\rho 1))))+\\
&\theta\nu(\kappa_4 + \kappa_3(2\xi - \Omega\kappa_4 + 2\nu\rho_1))\rho_2 = 0
\end{aligned}
\tag{2.34}
$$

$$
\begin{aligned}
&2\theta\xi^2\Omega\kappa_3 + 2\Omega^3\kappa_3^2+\\
&\Omega^2(4\kappa_3 + (1 - \theta)\kappa_2\kappa_3\kappa_4 - 2\theta\kappa_4^2 - 2\theta\nu\kappa_3\kappa_4\rho_1)+\\
&2\nu^2\kappa_3\rho_1\rho_3 + \xi(\theta\kappa_2 - 2\theta\Omega^2\kappa_3\kappa_4 + 2\nu\kappa_3\rho_3)+\\
&\xi\Omega((-2 + \theta)\kappa_2\kappa_3 + 2\theta\kappa_4 + 4\theta\nu\kappa_3\rho_1) + \nu(\theta\kappa_2\rho_1 + \kappa_4\rho_3)+\\
&\Omega(2 + (-1 - \theta)\kappa_2\kappa_4 + 2\theta\nu^2\kappa_3\rho_1^2)+\\
&\Omega\nu((-2 + \theta)\kappa_2\kappa_3\rho_1 + 2\theta\kappa_4\rho_1 - \kappa_3\kappa_4\rho_3) = 0
\end{aligned}
\tag{2.35}
$$

$$
\begin{aligned}
&2\theta\nu\xi^2\kappa_3 + 2\nu\Omega^2\kappa_3^2+\\
&2\theta\nu^2\kappa_4\rho_1 + 2\theta\nu^3\kappa_3\rho_1^2 + \nu(2 - \theta\rho_1)+\\
&\xi(-\theta + 2\theta\nu\kappa_4 + \Omega(\theta\kappa_3 - 2\theta\nu\kappa_3\kappa_4) + 4\theta\nu^2\kappa_3\rho_1)+\\
&\Omega(2\theta\kappa_4 - 2\theta\nu^2\kappa_3\kappa_4\rho_1 + \nu(4\kappa_3 - 2\theta\kappa_4^2 + \theta\kappa_3\rho_1)) = 0
\end{aligned}
\tag{2.36}
$$

With given values of the exogenous parameters, we can solve the coefficient of conjecture τ, ξ, φ, Ω, ν explicitly. The paths of onshore and offshore RMB exchange rates can be displayed after these coefficients are obtained. We will elaborate this part of work in Section 2.6.

2.5 Empirical Analysis

In this section, an empirical analysis of the two RMB exchange rates will be performed. There are two main purposes of the empirical analysis. First, to describe historical dynamic correlations between the two exchange rates and related economic variables. Second, to provide extra information including parameter setting for calibration of the theoretical model, in order to examine the extent to which the theoretical model captures features of the actual exchange rate behaviour.

The main data pertain to the onshore and offshore RMB exchange markets over the period from February 1^{st}, 2013 to May 31^{st}, 2017. On July 6^{th}, 2010, the PBoC and Bank of China (Hong Kong) Limited signed the agreement that banks in Hong Kong area can operate RMB trade independently. This agreement built foundation of the offshore RMB market. After two years, two distinct RMB exchange rates were formally established. In addition, on August 11^{th}, 2015, the PBoC re-adjusted the mechanism of CNY ("811 Reform of RMB Exchange Rate"). Therefore, the sample period incorporates the essential information for our research: the interactions between historical free-floating CNH and managed-floating CNY, and impact of "811 Reform of the RMB Exchange Rate".

2.5.1 Variables

The general information about the realized economic variables in compliance with those in the theoretical model, including offshore RMB exchange rate (CNH), onshore RMB exchange rate (CNY), RMB Non-Delivery Forward (HE), RMB Internet-Bank Forward (YE), Effective Exchange Rate Index of RMB (EI) and Reserves of Foreign Exchange (FX), is provided in this section.

2.5.1.1 Offshore (CNH) & Onshore (CNY) RMB Exchange Rate

$\tilde{e}_t$ (CNH) and e_t (CNY) are the key variables in this research. The daily data of CNH is the closing price cited from HKTMA. The daily data of CNY is the closing price from CFETC.

2.5.1.2 RMB Non-Delivery Forward (HE)

$E_t(\tilde{e}_{t+1})$, which is the aggregate offshore market expectation, can be reflected by the offshore RMB foreign exchange derivatives. The main derivatives include Non-delivery Forward (NDF), Non-delivery Options (NDO), CME RMB foreign exchange futures and options, and Non-delivery Swap (NDS). Among them, NDF is most often used to reflect the expectations for exchange rates. If the offshore market agents expect RMB to appreciate in the coming periods, they can purchase RMB and sell U.S. Dollar when the discount rate of the NDF declines, and then reverse the operation at a higher discount rate. The risk-free benefit can be obtained through matching the cash flows of two contracts with the same maturity date.

Terms of the NDF contract span from several months to years. The majority of NDF transaction concentrates on the contract with term less than 1 year. The contracts of 1-week, 1-month, 3-months, 6-months and 1-year have high levels of liquidity. Therefore, in the empirical analysis, the NDF 1-month is selected as the proxy of offshore market expectation of RMB exchange rate.

2.5.1.3 RMB Inter-Bank Forward (YE)

The onshore inter-bank RMB forward exchange rate is a suitable proxy variable of the onshore market expectation $E_t(e_{t+1})$ in the theoretical model. The onshore RMB forward rate is the exchange rate at a negotiated time at least one day after the transaction is completed. According to different dates of delivery, there are usually RMB forward exchange rates maturing in 1-day, 1-week, 1-month, 3-month, 6-month, 9-month and 1-year. In this study, onshore RMB 1-month forward rate, which is cited most frequently, is used as the representative of onshore inter-bank expectation.

2.5.1.4 Effective Exchange Rate Index of RMB (EI)

Currently, three major financial institutions issue effective exchange rate indexes: the Bank for International Settlements (BIS), the European Central Bank (ECB) and the Federal Reserve of U.S.

(FD). The main idea of compiling an effective exchange rate index is the weighted exchange rate of domestic currency against foreign currencies based on bilateral international trading amounts.

Among these three indices, the index complied by BIS is the most authoritative. Therefore, it is used as proxy indicator for the effective exchange rate of RMB in the theoretical model $\bar{e}_t$. The formula of the BIS effective exchange rate index is:

$$NI_{BIS,i,t} = \prod_{j=1}^{N} (e_{i,j,t})^{W_j}\,; \tag{2.37}$$

$$RI_{BIS,i,t} = \prod_{j=1}^{N} \left(\frac{P_{i,t}}{P_{j,t}} e_{i,j,t}\right)^{W_j} \tag{2.38}$$

where $NI_{BIS,i,t}$ is the nominal effective exchange rate index of domestic currency i at time t. j is the index of the foreign country. $RI_{BIS,i,t}$ is the corresponding real effective exchange rate index at time t. $e_{i,j,t}$ represents the nominal exchange rate between domestic country i and foreign country j at time t. W_j is the weight of currency j in the currency basket. $P_{i,t}$ ($P_{j,t}$) is the price level of domestic (foreign) country at time t.

BIS also provides the formula to compute the trading weight W_j. W_j can be calculated as:

$$W_j = \left(\frac{M_i}{M_i + X_i}\right) W_j^m + \left(\frac{X_i}{M_i + X_i}\right) W_j^x \tag{2.39}$$

where $M_i = \sum_{j=1}^{N} M_{i,j}$ is the aggregate amount of country i's imports; $X_i = \sum_{j=1}^{N} X_{i,j}$ is the aggregate amount of country i's exports. $W_j^m(W_j^x)$ represents the trading weight of importing from (exporting to) country j.

2.5.1.5 Reserves of Foreign Exchange (FX)

The foreign exchange reserves are the foreign assets held by the domestic country. The foreign exchange reserves are an important constitution of international liquidity, playing an essential role in regulating the account balance, intervening in the foreign exchange market and stabilizing the exchange rate, especially coping with financial crise. China has maintained a trading surplus during the past 40 years and foreign exchange reserve has been accumulated simultaneously. At the end of June 2011, China's foreign exchange reserves reached US $318.11 Billion, ranking first around the world. The foreign exchange reserves reflect the ability of monetary authority to intervene in the foreign exchange market. Too little foreign exchange reserve will affect the confidence of stability of domestic currency. Generally, the central bank operates through purchasing or selling foreign exchange reserves to adjust the supply and demand of foreign exchange, and ease the fluctuations of the exchange rate or change its direction. In this empirical research, the monthly foreign exchange reserve provided by the PBoC after daily smoothing is used as a proxy variable of foreign exchange $\overline{FX}_{t-1}$ in the theoretical model.

2.5.2 Empirical Methodology

As can be seen from the properties described in the previous section, these variables interact with each other and exhibit different motion patterns over time. Therefore, based on these properties, and in order to better compare with the sequences obtained in simulation section, a methodology of empirical analysis in a time series framework is illustrated in this section.

2.5.2.1 ADF Test

Before establishing a time series model, unit-root tests are required to verify whether the time series investigated are stationary or not. In general, the DF test, the ADF test and the PP test are the three main methodologies. In our research, ADF (Augmented Dickey-Fuller) test which is used most frequently, is used to determine the stationary of time series. The formula for ADF test is:

$$\Delta Y_t = \beta_0 + \beta_1 t + \delta Y_{t-1} + \sum_{i=1}^{m} \delta_i \Delta Y_{t-1} + \epsilon_t \tag{2.40}$$

where ϵ_t is the white noise error term. ADF test examines the significance of coefficient δ. The null hypothesis is $\delta = 0$, which corresponds to the existence of unit-root, indicating that the series is not stationary. The alternative hypothesis is $\delta < 0$ implying that the time series is stationary.

2.5.2.2 Co-Integration Test

Co-integration means that the linear combination of two or more non-stationary time series is stationary. Co-integration implies a long-term or balanced relationship among the series. In co-integration analysis, the individual time series have to be one order integrated $I(1)$, which means stationary after first order difference. Additionally, statistics of LR test, AIC, SC and HQ information criteria can be used to determine the optimal lag order of the model. Moreover, the main co-integration test mainly consists of two-step test for a single equation, EG test, and Johansen co-integration test for vector auto-regressive model. Since multi variables are included in our empirical analysis, the Johansen co-integration test is used.

Co-integration likelihood ratio test method proposed by Johansen mainly includes the trace test method and the maximum eigenvalue test method. The null hypothesis of trace test H_0 is at most r co-integration exist. The non-null hypothesis H_1 is that m co-integration exist (full rank). The statistics is written as:

$$LR_{tr}(r|m) = -T \sum_{i=r+1}^{m} log(1 - \lambda_i) \tag{2.41}$$

where λ_i is the i^{th} eigen-value ranked in descending order. T is the number of total periods of observation.

2.5.2.3 VECM

The Vector Auto-regressive (VAR) model is usually used to capture the long-term movement of multi-variables. The general form of a VAR is as follows:

$$\vec{y}_t = \vec{\psi}_t + \sum_{i=1}^{p} \vec{\varphi}_p \vec{y}_{t-i} + \vec{a}_t \tag{2.42}$$

where $\vec{y}_t$ is vector of k-dimension stationary endogenous variable. $\vec{\psi}_t = \vec{\psi}_0 + \vec{\psi}_1 t$, both $\vec{\psi}_0$ and $\vec{\psi}_1$ are k-dimension vector, and $\vec{\varphi}_p$ is a $k \times k$ matrix $\forall\ i>0$. $\vec{a}_t$ is an error vector which is assumed as a $i.i.d$ random sequence, of which the mean is 0 and co-variance matrix $\Sigma_{\vec{a}_t}$ is positive semi-definite.

The general form of the corresponding error-correction model of VAR(p) (VECM) is viewed as VAR model that imposes co-integration constraints on variables, and can only be used for modeling time series with co-integration. The VECM is written as follow:

$$\Delta\vec{y}_t = \vec{\psi}_t + \vec{\Pi}\vec{y}_{t-1} + \sum_{i=1}^{p-1} \vec{\Phi}_i^{\star} \Delta\vec{y}_{t-i} + \vec{a}_t \tag{2.43}$$

This model is proposed by [47]. For the co-integrated variables y_1 and y_2, their short-term non-equilibrium relationship can always be expressed by an error correction model. In above formula, $\vec{\Pi}\vec{y}_{t-1}$ is the item of deviation to long-term equilibrium. $\vec{\Pi}$ is a short-term adjustment parameter. VECM can be used to study short-term deviations from the long-term equilibrium.

2.5.2.4 Granger Causality Test

The Granger Causality Test is used frequently to analyze the causality relationship between economic variables. The null hypothesis of Granger Causality Test is the current value of variable X is not affected by current and lagged values of another variable Y.

The general form of Granger Causality Test is:

$$X_t = \alpha_{1,0} + \sum_{i=1}^{n} \alpha_{1,i} X_{t-i} + \sum_{j=1}^{n} \beta_{1,j} Y_{t-j} + \epsilon_{1t} \tag{2.44}$$

$$Y_t = \alpha_{2,0} + \sum_{i=1}^{n} \alpha_{2,i} X_{t-i} + \sum_{j=1}^{n} \beta_{2,j} Y_{t-j} + \epsilon_{2t} \tag{2.45}$$

The specification of equations (2.44) and (2.45) indicates that the Granger Causality Test is a kind of vector auto-regressive model. If not all $\beta_{1,j}$ equal to zero, then variable Y is considered as uni-directional Granger cause to variable X. If not all $\alpha_{2,i}$ equal to zero, then variable X is considered as uni-directional Granger cause to variable Y. If not all $\beta_{1,j}$ and $\alpha_{2,i}$ equal to zero simultaneously, then a bi-directional causality relationship exists between variable X and Y. In general, an F-statistic is constructed in Granger Causality Test. Through comparing the estimated value of F-statistic with the critical value, the Granger Causality relationship can be determined.

2.5.3 Descriptive Statistics

The statistics of variables fitted in the empirical analysis are summarized in the following Table 2.1. CNH, CNY, HE and YE are daily data, while EI and FX are monthly data. Here, the value of EI and FX are assumed to be unchanged in each month. Additionally, the scales of EI and FX have also been adjusted in order to transform the coefficients to same order of magnitudes as other variables. Specifically, EI is divided by 1,000, and FX is divided by 100,000.

Table 2.1 Descriptive Statistics Summary of Variables

Variable	Mean	Std.Dev.	Max	Min	Kurtosis	Skewness
CNH	0.157	0.006	0.166	0.143	0.940	-0.672
CNY	0.157	0.006	0.165	0.144	-0.790	-0.767
EI	0.118	0.053	0.128	0.108	-0.124	0.232
HE	0.157	0.007	0.164	0.142	-0.889	-0.773
YE	0.157	0.006	0.165	0.144	-0.760	-0.770
FX	0.352	0.032	0.399	0.300	-1.364	-0.172

The statistical properties of exchange spot and forward rates are quite similar, then the evolution of $(CNH_t, CNY_t, HE_t, YE_t)$ is intuitively considered as an endogenous system, which however needs to be verified by Johansen co-integration test. Additionally, EI_t and FX_t are treated as exogenous series, which is in consistent with the setting in the theoretical model. Moreover, the ADF test is used to test whether the series in Table 2.1 are stationary or not. The p-values of the test reject the null hypothesis that the series are stationary. However, their first-order difference series $(x_t - x_{t-1})$ are stationary, namely, all the six series are $I(1)$ process. The corresponding descriptive statistics are summarized in Table 2.2.

Table 2.2 Descriptive Statistics Summary of Differentiated Variables

Variable	Mean	Std.Dev.	Max	Min	Kurtosis	Skewness
dCNH	-3.250e-04	9.273e-03	9.273e-03	-1.330e-02	8.941	-1.393
dCNY	-3.450e-05	6.752e-04	5.164e-03	-7.983e-03	23.941	-1.208
dEI	2.622e-05	2.290e-04	9.898e-04	-8.876e-04	2.334	6.756e-01
dHE	-3.373e-05	9.037e-04	6.503e-03	-1.772e-02	162.077	-8.493
dYE	-3.965e-05	7.096e-04	3.860e-03	-1.323e-02	112.406	-6.458
dFX	-4.054e-05	2.875e-04	1.293e-03	-2.431e-02	6.314	-2.390e-01

Note: all the first order difference series are logarithmized.

2.5.4 Result of Estimation

In this section, the interactions of CNH and CNY exchange rates are investigated. VECM, which is used to describe the long-term interaction, is adopted to investigate the dynamics of two RMB exchange rate series.

2.5.4.1 VECM of Bi-Variate System Feb 2013—Aug 2015

Figure 2.1 describes the evolution of the sample CNH and CNY series, in which August 11^{th}, 2015 works as a threshold, when the PBoC announced the new mechanism of RMB exchange rate. Before the threshold, both the CNY and CNH series were at a relatively high level, which indicates U.S. Dollar was weak against RMB. After the threshold, both exchange rate series kept declining, which implies a persistent appreciation process of U.S. Dollar against RMB. Therefore, the historical exchange rate series are estimated separately due to the different behavioural patterns in each sub-period.

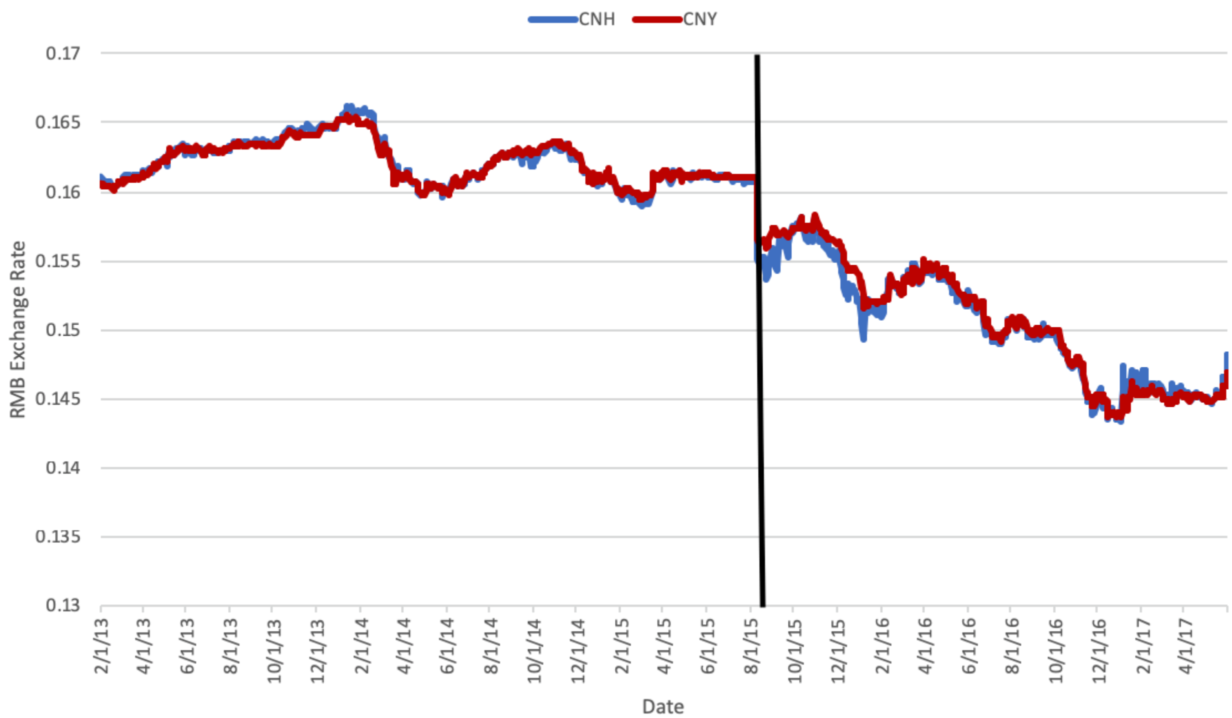


Figure 2.1 Historical CNH & CNY 2013—2017

First of all, the logarithmic exchange rates from February 1^{st}, 2013 to August 10^{th}, 2015 are selected. The Johansen test rejects the null hypothesis of no co-integration, which indicates that a co-integration vector is present in the bi-variate system of CNH and CNY exchange rates. The estimated co-integration vector is (1,-1.080) with p-value less than 0.01. Therefore, the relationship of long-term co-integration between two exchange rates is:

$$CNH_t = 1.080 CNY_t \tag{2.46}$$

Equation (2.46) indicates that the offshore RMB exchange rate will appreciate (depreciate) 1.080% when the onshore exchange rate appreciate (depreciate) 1%. It is notable that the estimated co-integration vector is very close to (1, -1) which suggests that the two exchange rates behave in a similar way. Figure 2.2 describes the co-integrated series between the two exchange rates which is stationary and fluctuates around zero.

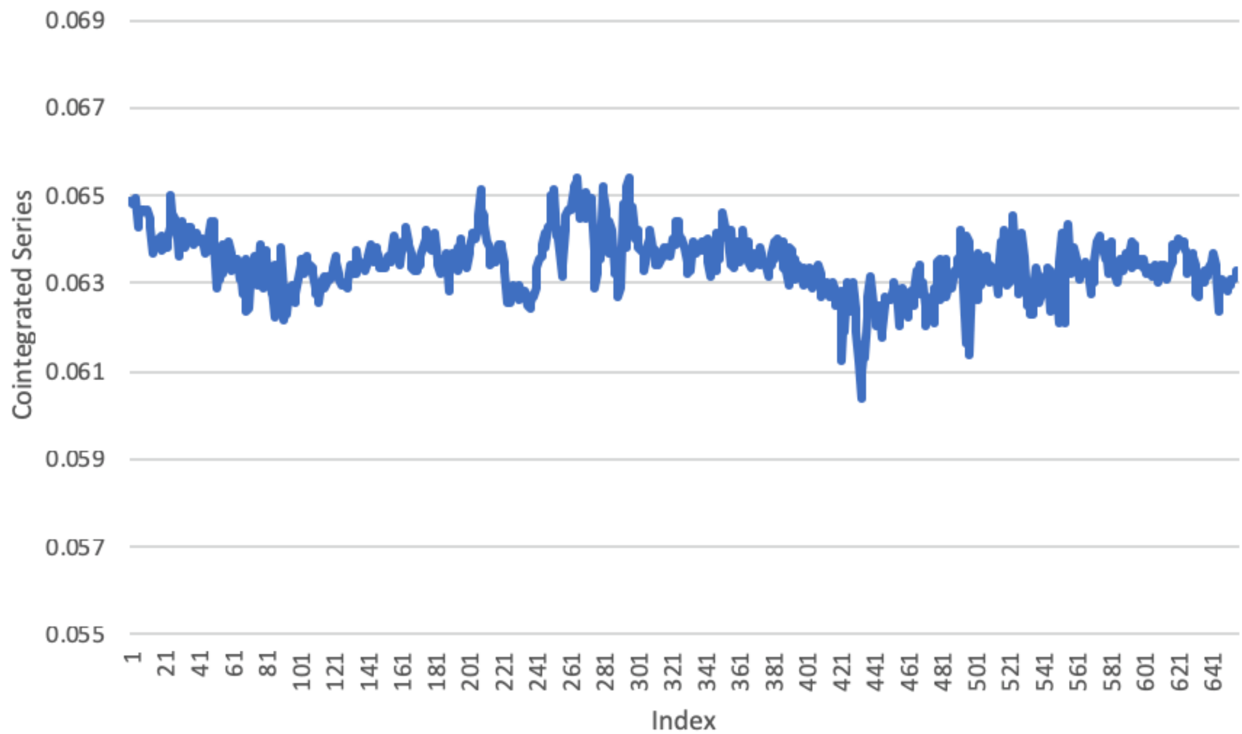


Figure 2.2 Time Plot of Co-integrated Series between Onshore & Offshore Exchange Rates Feb 2013—Aug 2015

The specification of bi-variate (CNH_t, CNY_t) VECM is (2.47) and (2.48). The error correction term EC_{t-1} is $CNH_t - 1.080CNY_t$, and the lagging order $p = 3$ is selected by information criteria (AIC, BIC & HQ).

$$\Delta CNH_t = \psi_1 + \Pi_1 EC_{t-1} + \Phi_{1,11}\Delta CNH_{t-1} + \Phi_{1,12}\Delta CNY_{t-1} + \Phi_{2,11}\Delta CNH_{t-2} + \Phi_{2,12}\Delta CNY_{t-2} + \epsilon_{1t} \tag{2.47}$$

$$\Delta CNY_t = \psi_2 + \Pi_2 EC_{t-1} + \Phi_{1,12}\Delta CNH_{t-1} + \Phi_{1,22}\Delta CNY_{t-1} + \Phi_{2,12}\Delta CNH_{t-2} + \Phi_{2,22}\Delta CNY_{t-2} + \epsilon_{2t} \tag{2.48}$$

Table 2.3 Estimation of Bi-Variate CNH_t & CNY_t VECM Feb 2013—Aug 2015

Parameters	VECM(3)	
	(1-1)	(1-2)
	ΔCNH_t	ΔCNY_t
Constant	0.007	-0.003
	(0.002)***	(0.002)
EC_{t-1}	-0.110	0.044
	(0.032)***	(0.027)
ΔCNH_{t-1}	-0.030	0.255
	(0.052)	(0.061)***
ΔCNH_{t-2}	-0.036	-0.107
	(0.044)	(0.051)***
ΔCNY_{t-1}	0.227	-0.106
	(0.049)***	(0.058)*
ΔCNY_{t-2}	0.123	-0.194
	(0.042)*	(0.049)***
$Adj.R^2$	0.07	0.06

Note: Robust standard errors in parentheses *** p <0.01, ** p <0.05, * p <0.1.

The estimation results for the first stage are summarized in Table 2.3.

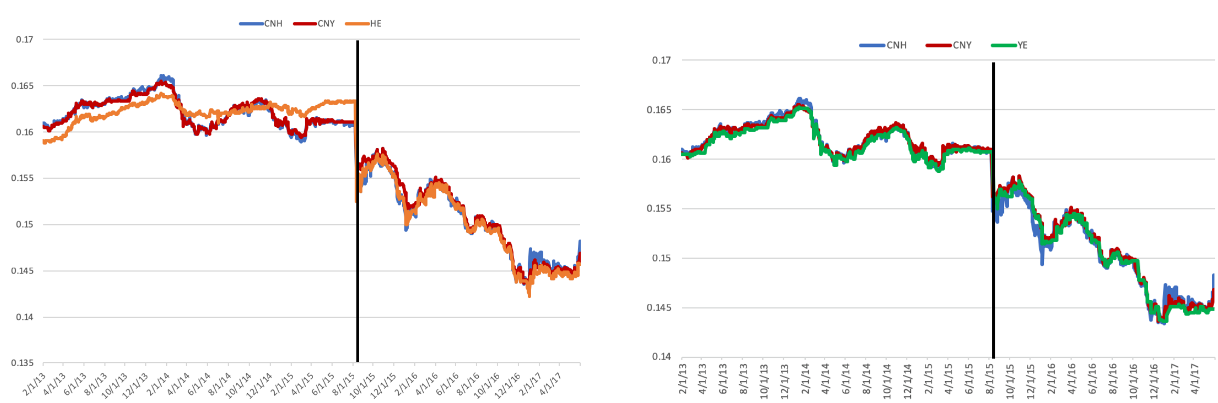
Different features of equilibrium correction mechanism have been detected for the onshore and offshore exchange rates. The return of the offshore exchange rate is significantly affected by the error correction term EC_{t-1} while the return of the onshore exchange rate is not. In addition, the cross-effects are found between the two exchange rates. Both the first and second order lagged returns of CNY have significant positive (0.227, 0.123) marginal effects on the return of CNH.

However, the lagged items of CNH themselves do not significantly impact the return of CNH. On the other hand, the first and second order lagged returns of CNH have positive (0.255) and negative (-0.107) impacts on the return of CNY, while the lagged returns of CNY have negative (-0.101, -0.194) impacts on the return of CNY itself.

The adjusted R^2 in Table 2.3 suggests that the proportion of variation explained by the bi-variate (CNH_t, CNY_t) system is low (0.07, 0.06). The evolution of exchange rates may be influenced by additional economic variables.

2.5.4.2 VECM of Multi-Variate System with Exogenous Variables Feb 2013—Aug 2015

The forward rates typically correlate with the spot rates. Two historical exchange rates and the corresponding onshore and offshore market forwards (HE,YE) are described in the two plots of Figure 2.3. The forward series intertwine with the exchange rate series closely which supports the rational expectation setting in the theoretical model.



(a) CNH, CNY & HE

(b) CNH, CNY & YE

Figure 2.3 Historical Spot & Forward RMB Exchange Rates

The Johansen test is applied to investigate the long-term equilibrium relationship of multi-variate system $(CNH_t, CNY_t, HE_t, YE_t)$. The test rejects the null hypothesis that the two exchange rates are not co-integrated with the forward series. However, the test result suggests that co-integrated relationships exist between the two spot RMB exchange rate series and the corresponding forward rates. The estimated co-integration vectors are (1, 0.054, 0.015, -1.096) and (1, -1.777, 0.071, 0.641) respectively, both of which are significant with p-value less than 0.01.

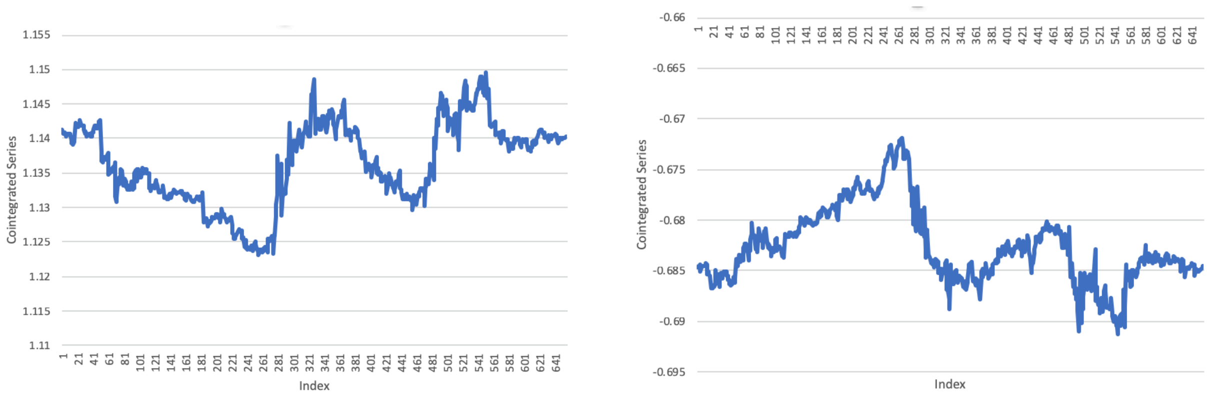
Figure 2.4 displays the time plot of the two co-integrated series, both of which show the characteristics of stationary time series as expected.

$$CNH_t = -0.054CNY_t - 0.015HE_t + 1.096YE_t \tag{2.49}$$

$$CNH_t = 1.777CNY_t - 0.071HE_t - 0.641YE_t \tag{2.50}$$

Equations (2.49) and (2.50) display two mechanisms of long-term adjusting. In the first equation, the offshore exchange rate will decrease 0.054% if onshore exchange rate increases 1%. When the offshore forward rate decrease 1%, the offshore exchange rate will increase 0.015%. If the onshore inter-bank forward rate increases 1%, the offshore exchange rate will increase 1.096%. In the second equation, the offshore exchange rate increases 1.777% if the onshore exchange rate increases 1%. When the offshore forward rate decreases 1%, the offshore exchange rate will increase 0.071%. If the onshore inter-bank forward rate increases 1%, the offshore exchange rate will decrease 0.641%.

Additionally, the exogenous effective exchange rate index (EI) and foreign exchange reserve (FX) which are the essential components in the model of onshore exchange rate are added as well. In the left plot of Figure 2.5, the moving pattern of EI is different before and after August 11^{th}, 2015 which works as the threshold. In the former stage, CNH and CNY declined from a high level, while



(a) Cointegrated Series 1

(b) Cointegrated Series 2

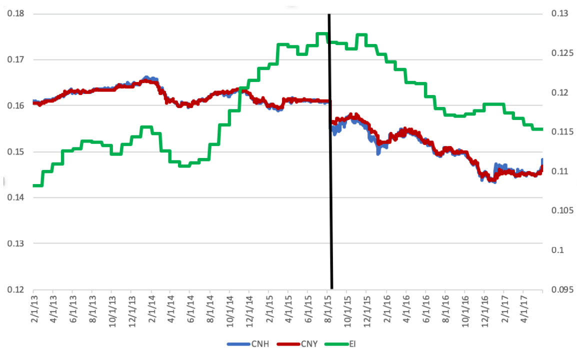
Figure 2.4 Time Plot of Co-integrated Series between Spot & Forward Exchange Rates Feb 2013—Aug 2015

EI underwent a soaring process. In the latter stage, CNY, CNH and EI moved in a similar trend. During this period, U.S. Dollar appreciated with RMB and EI declined simultaneously.

Moreover, as seen from the right plot of Figure 2.5, the series of foreign exchange reserves and two exchange rates maintain the similar moving pattern in both stages. In the former period of U.S. Dollar depreciation, foreign exchange reserves increased, while in the latter period of U.S. Dollar appreciation, foreign exchange reserves decreased. This is in consistent with the description of the PBoC smoothing the change of exchange rate through adjusting its foreign exchange reserves in the theoretical model.

The specification of multi-variate $(CNH_t, CNY_t, HE_t, YE_t)$ VECM with exogenous variables EI_t and FX_t is described in equations (2.51) to (2.54).

The error correction items are $EC_{1,t-1}$ and $EC_{2,t-1}$ equalling $CNH_t + 0.054CNY_t + 0.015HE_t - 1.096YE_t$ and $CNH_t - 1.777CNY_t + 0.071HE_t + 0.641YE_t$, respectively. Similarly, the lagging order $p = 3$ is also selected by the information criteria.



(a) CNH, CNY & EI

(b) CNH, CNY & FX

Figure 2.5 Historical RMB Exchange Rates & Effective Exchange Rate Index, Foreign Exchange Reserves Feb 2013—Aug 2015

$$\begin{aligned}\Delta CNH_t = \psi_1 + \Pi_{11}EC_{1,t-1} + \Pi_{12}EC_{2,t-1} + \Phi_{1,11}\Delta CNH_{t-1} + \Phi_{1,12}\Delta CNY_{t-1} + \\ \Phi_{1,13}\Delta HE_{t-1} + \Phi_{1,14}\Delta YE_{t-1} + \Phi_{2,11}\Delta CNH_{t-2} + \Phi_{2,12}\Delta CNY_{t-2} + \\ \Phi_{2,13}\Delta HE_{t-2} + \Phi_{2,14}\Delta YE_{t-2} + \beta_{11}\Delta EI_t + \beta_{12}\Delta FX_t + \epsilon_{1t}\end{aligned} \tag{2.51}$$

$$\begin{aligned}\Delta CNY_t = \psi_2 + \Pi_{21}EC_{1,t-1} + \Pi_{22}EC_{2,t-1} + \Phi_{1,21}\Delta CNH_{t-1} + \Phi_{1,22}\Delta CNY_{t-1} + \\ \Phi_{1,23}\Delta HE_{t-1} + \Phi_{1,24}\Delta YE_{t-1} + \Phi_{2,21}\Delta CNH_{t-2} + \Phi_{2,22}\Delta CNY_{t-2} + \\ \Phi_{2,23}\Delta HE_{t-2} + \Phi_{2,24}\Delta YE_{t-2} + \beta_{21}\Delta EI_t + \beta_{22}\Delta FX_t + \epsilon_{2t}\end{aligned} \tag{2.52}$$

$$\begin{aligned}\Delta HE_t = \psi_3 + \Pi_{31}EC_{1,t-1} + \Pi_{32}EC_{2,t-1} + \Phi_{1,31}\Delta CNH_{t-1} + \Phi_{1,32}\Delta CNY_{t-1} + \\ \Phi_{1,33}\Delta HE_{t-1} + \Phi_{1,34}\Delta YE_{t-1} + \Phi_{2,31}\Delta CNH_{t-2} + \Phi_{2,32}\Delta CNY_{t-2} + \\ \Phi_{2,33}\Delta HE_{t-2} + \Phi_{2,34}\Delta YE_{t-2} + \beta_{31}\Delta EI_t + \beta_{32}\Delta FX_t + \epsilon_{3t}\end{aligned} \tag{2.53}$$

$$\Delta YE_t = \psi_4 + \Pi_{41}EC_{1,t-1} + \Pi_{42}EC_{2,t-1} + \Phi_{1,41}\Delta CNH_{t-1} + \Phi_{1,42}\Delta CNY_{t-1} + \Phi_{1,43}\Delta HE_{t-1} + \Phi_{1,44}\Delta YE_{t-1} + \Phi_{2,41}\Delta CNH_{t-2} + \Phi_{2,42}\Delta CNY_{t-2} + \Phi_{2,43}\Delta HE_{t-2} + \Phi_{2,44}\Delta YE_{t-2} + \beta_{41}\Delta EI_t + \beta_{42}\Delta FX_t + \epsilon_{4t} \tag{2.54}$$

The estimation results for the first stage are summarized in Table 2.4 below.

Setting the non-significant estimated values as zero, and the coefficients of error correction items Π can be refined as:

$$\Pi' = \begin{bmatrix} -0.108 & 0 & 0 & 0.182 \\ -0.068 & 0.031 & 0 & -0.050 \end{bmatrix} \tag{2.55}$$

This specific structure shows that the return of offshore exchange rate correlates with both the first and second error correction items negatively (-0.108, -0.068) while the return of onshore exchange rate only correlates with the second error correction item positively (0.031). In addition, both error correction items do not have significant impact on the return of offshore forward rate. Moreover, the onshore inter-bank forward rates correlate with the first error correction item positively (0.182) while correlates with the second error correction item negatively (-0.050). The results in both Table 2.3 and 2.4 suggest that the adjusting velocity of the offshore exchange rate is faster than that of the onshore exchange rate which indicates the higher sensitivity of the offshore exchange market.

Column (2-1) of Table 2.4 shows that the first order lagged return of onshore exchange rate has a significant positive (0.257) marginal impact on the return of offshore exchange rate, indicating the previous return of the onshore exchange rate drives the current offshore exchange rate to move in the same direction. This result confirms that participants in the offshore exchange market refer to the onshore exchange rate to make their trading decisions. In addition, the first order lagged return of onshore inter-bank forward return impacts the return of offshore exchange rate negatively (-0.065). Moreover, the first order lagged return of offshore forward rate has a significant positive impact (0.110) on the return of offshore exchange rate which implies that the offshore forward rate is a leading indicator of the spot rate.

Column (2-2) in the table suggests that both the first and second order lagged returns of onshore and offshore exchange rates have influence on the return of onshore exchange rate. Specifically, the impacts from returns of offshore exchange rates are positive (0.210, 0.082), which confirms that commercial banks in the onshore inter-bank market incorporate the offshore exchange rate in their value function of transaction. In addition, the impacts from the onshore exchange rates are negative (-0.143, -0.244) which reflects the auto-regression effect on onshore exchange rate. The PBoC inhibits the uni-directional movement of the onshore exchange rate.

Besides this, the second order lagged return of onshore inter-bank forward rates has a significant positive (0.093) influence on the return of the onshore exchange rate. The onshore inter-bank forward rate is a leading indicator of the spot rate, which reflects the expectation of onshore market agents. Moreover, the exogenous effective exchange rate index also exerts a positive (0.161) impact on the onshore exchange rate. Unlike the offshore market agents, the onshore market agents use the effective exchange rate index as determinant for onshore exchange rate. It is notable that the influence from foreign reserves is not significant, which implies that the PBoC may has not performed intervention frequently in this period.

Column (2-3) provides information on the offshore market expectation. Both first and second order lagged returns of offshore exchange rate have positive (0.296, 0.077) effects on the return of offshore forward rate, which confirms that the chartists apply trend-following strategy to generate their expectation. On the other hand, both the first and second returns of onshore exchange rate have negative (-0.112, -0.097) impacts on the return of offshore forward rate, which reflects the offshore exchange rate converges to the onshore exchange rate in fundamentalist's expectation. Besides this, the first order lagged return of onshore forward rate has a significant positive (0.031) effect on the return of offshore forward rate, which confirms the influence from onshore inter-bank forward rate on the onshore spot rate transfers to the offshore forward exchange rate again. Moreover, the first and second order lagged offshore forward rates have significant negative (-0.255, -0.088) auto-correlation effects, which reflects the characteristics of path dependence.

In Column (2-4), the return of onshore inter-bank forward rate is positively (0.301, 0.250) correlated with both the first and second returns of onshore exchange rate. Besides this, the first order return of onshore inter-bank forward rate has a negative (-0.211) auto-correlated effect, indicating the feature of path dependence as well. Furthermore, the comparison between Tables 2.3 and 2.4 shows that the adjusted R^2 of CNH case increases from 0.07 to 0.18, and that of CNY case has increased from 0.06 to 0.17 with additional explanatory variables, which suggests the additional explanatory variables significantly improve the model's accuracy.

The Granger Causality test provides an alternative approach to investigate the causal relationships between the returns of two exchange rates and that of other economic variables. The results are shown in Table 2.5.

The results in Table 2.5 reject the null hypothesis that no causality relationship exists between the returns of offshore and onshore exchange rates. They Granger cause each other. In addition, the return of offshore exchange rate and return of offshore forward rate are Granger causes to each other as well. Besides this, there is uni-directional Granger cause from the return of onshore inter-bank forward to the return of offshore exchange rate. Moreover, the uni-directional causality runs from the return of onshore exchange rate to the inter-bank forward rate, which confirms that the onshore market agents generate their expectation based on the current spot rate. Furthermore, the return of the effective exchange rate index is also uni-directional Granger cause to return of onshore exchange rate which indicates that the effective exchange rate index is a key component of onshore exchange rate determination. Overall, the Granger causality test results are in line with the estimation results shown in Table 2.4.

2.5.4.3 VECM of Bi-Variate System Aug 2015—May 2017

For the second regime August 11^{th}, 2015-May 31^{st}, 2017, the Johansen test rejects the null hypothesis of no co-integration between the two exchange rates. Therefore, a co-integration vector exists in the bi-variate system of onshore and offshore exchange rates as well. The estimated co-integration vector is (1, -0.917) with significant p-value less than 0.02, then the long-term equilibrium relationship between two exchange rates is written as:

$$CNH_t = 0.917CNY_t \tag{2.56}$$

Equation (2.56) indicates that the offshore exchange rate will appreciate (depreciate) 0.917% when the onshore exchange rate appreciates (depreciates) 1%. Although the vector is still close to (1, -1) implying that the two exchange rates move in a similar pattern, the influence of the onshore exchange rate on the offshore exchange rate is weakened, compared with the equilibrium relationship of the two exchange rates in the first stage (1.080%). Figure 2.6 exhibits the corresponding co-integrated series which fluctuates around zero and shows the feature of stationary series as well.

Similarly, the VECM is applied to capture the dynamic correlation of the two exchange rates in this period. The results of estimation are summarized in Table 2.6. The information criteria selects the lagging order p as 4.

Table 2.6 shows that only the onshore exchange rate is affected by the error correction item significantly in the second stage, while the offshore exchange rate is not influenced by the error correction item significantly. This result is opposite to that of Table 2.3.

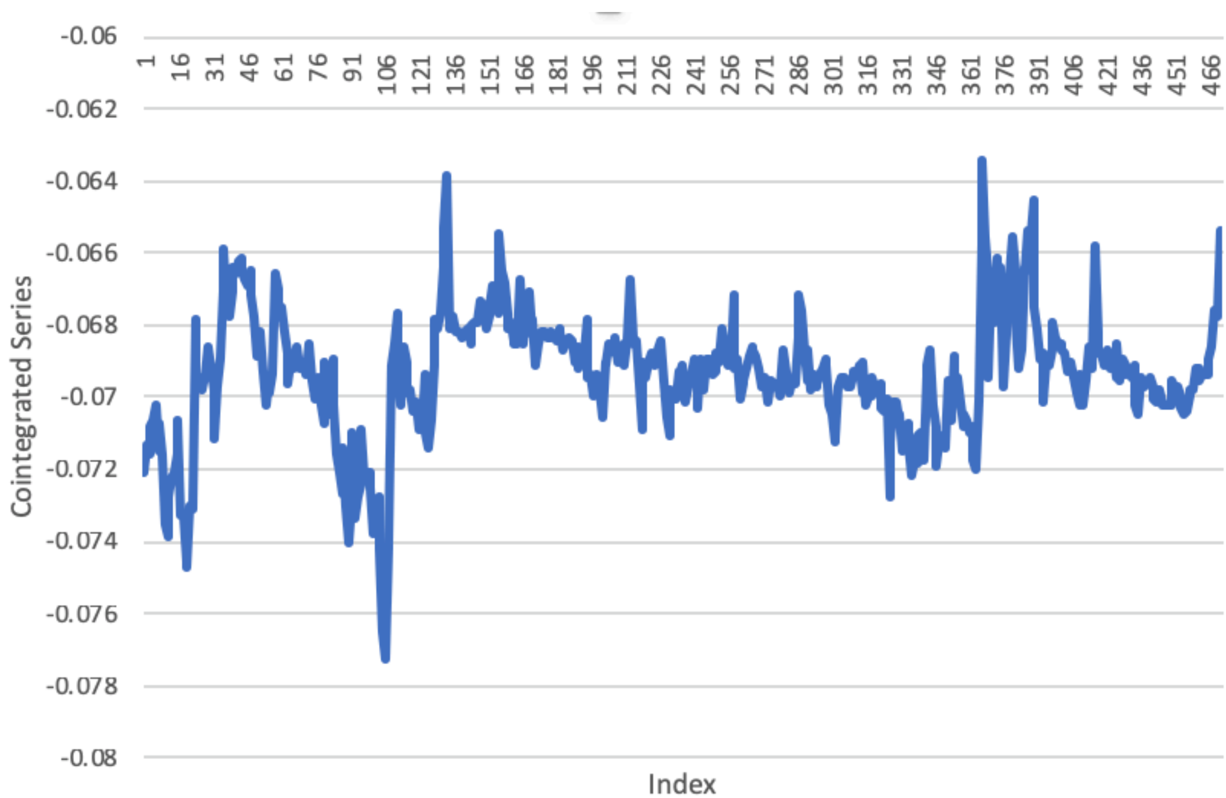


Figure 2.6 Time Plot of Co-integrated Series between the Onshore & Offshore Exchange Rate Aug 2015—May 2017

Column (3-1) of Table 2.6 suggests that only the second order lagged item of onshore exchange rate return has a significant negative cross-effect (-0.135). Compared with column (1-1) in Table 2.3, the influence from return of the onshore exchange rate on that of offshore exchange rate decreases, and the sign is changed as well.

Column (3-2) of the table displays that both the first and second order lagged returns of onshore exchange rate have significant negative impacts (-0.161, -0.180) on the return of onshore exchange rate. Additionally, only the first order lagged offshore exchange rate return has a positive effect (0.152) on that of the onshore exchange rate.

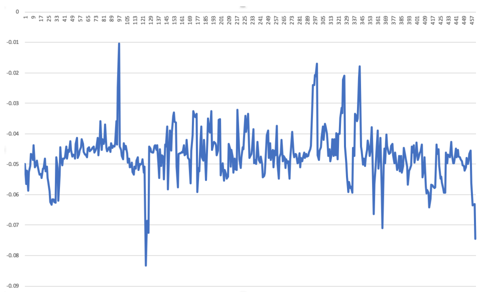
Compared with (1-2) in Table 2.3, only the first order lagged offshore exchange rate return has an impact on the return of onshore exchange rate in the second stage.

The adjusted R^2 in Table 2.6 suggests that the proportion of variation explained by the bi-variate (CNH_t, CNY_t) system is (0.02, 0.09). Compared to the results in Table 2.3, the adjusted R^2 of offshore exchange rate case declines 0.05, while that of onshore exchange case increases 0.03. It indicates that some other factors in addition to the onshore exchange rate perform more significant impacts on the determination of offshore exchange rate. On the other hand, the significant long-term equilibrium relationship implies the weight of offshore exchange rate becomes higher in determination of the onshore exchange rate .

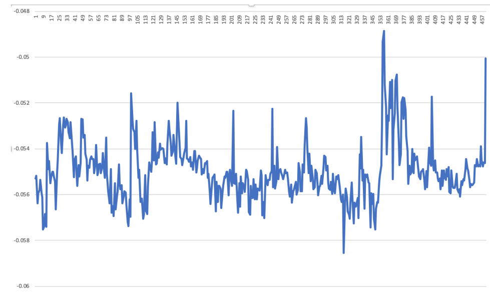
2.5.4.4 VECM of Multi-Variate System with Exogenous Variables Aug 2015—May 2017

Similarly, the Johansen test is applied to investigate the long-term equilibrium relationship of multi-variate system $(CNH_t, CNY_t, HE_t, YE_t)$ in the second stage. The first 10 daily observations are removed in order to eliminate the high volatility at the initial stage after the reform. The test rejects the null hypothesis that the spot exchange rates are not co-integrated with the forward series. However, three co-integrated relationships are present between the spot exchange rates and forward rates, which indicates that a unique long term equilibrium does not exist. In Figure 2.7, the three co-integrated series display the features of stationary time series.

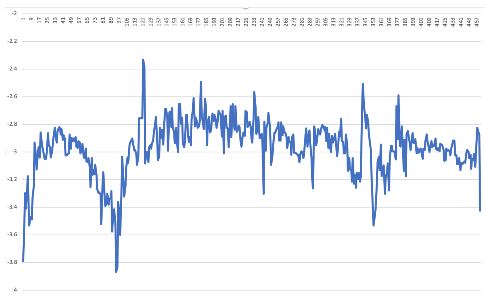
The specification of the multi-variate $(CNH_t, CNY_t, HE_t, YE_t)$ VECM with EI_t and FX_t used to estimate the dynamics correlation of the second stage is described in equations (2.57) to (2.60). The order is selected as $p = 4$ by the information criteria.



(a) Cointegrated Series 1



(b) Cointegrated Series 2



(c) Cointegrated Series 3

Figure 2.7 Time Plot of Co-integrated Series between Spot & Forward Exchange Rates Aug 2015—May 2017

$$
\begin{aligned}
\Delta CNH_t = \psi_1 &+ \Pi_{11} EC_{1,t-1} + \Pi_{12} EC_{2,t-1} + \Pi_{13} EC_{3,t-1} + \\
&\Phi_{1,11} \Delta CNH_{t-1} + \Phi_{1,12} \Delta CNY_{t-1} + \Phi_{1,13} \Delta HE_{t-1} + \Phi_{1,14} \Delta YE_{t-1} + \\
&\Phi_{2,11} \Delta CNH_{t-2} + \Phi_{2,12} \Delta CNY_{t-2} + \Phi_{2,13} \Delta HE_{t-2} + \Phi_{2,14} \Delta YE_{t-2} + \\
&\Phi_{3,11} \Delta CNH_{t-3} + \Phi_{3,12} \Delta CNY_{t-3} + \Phi_{3,13} \Delta HE_{t-3} + \Phi_{3,14} \Delta YE_{t-3} + \\
&\beta_{11} \Delta EI_t + \beta_{12} \Delta FX_t + \epsilon_{1t}
\end{aligned} \tag{2.57}
$$

$$\begin{aligned}\Delta CNY_t = \psi_2 + \Pi_{21}EC_{1,t-1} + \Pi_{22}EC_{2,t-1} + \Pi_{23}EC_{3,t-1}+ \\ \Phi_{1,21}\Delta CNH_{t-1} + \Phi_{1,22}\Delta CNY_{t-1} + \Phi_{1,23}\Delta HE_{t-1} + \Phi_{1,24}\Delta YE_{t-1}+ \\ \Phi_{2,21}\Delta CNH_{t-2} + \Phi_{2,22}\Delta CNY_{t-2} + \Phi_{2,23}\Delta HE_{t-2} + \Phi_{2,24}\Delta YE_{t-2}+ \\ \Phi_{3,21}\Delta CNH_{t-3} + \Phi_{3,22}\Delta CNY_{t-3} + \Phi_{3,23}\Delta HE_{t-3} + \Phi_{3,24}\Delta YE_{t-3}+ \\ \beta_{21}\Delta EI_t + \beta_{22}\Delta FX_t + \epsilon_{2t}\end{aligned} \tag{2.58}$$

$$\begin{aligned}\Delta HE_t = \psi_3 + \Pi_{31}EC_{1,t-1} + \Pi_{32}EC_{2,t-1} + \Pi_{33}EC_{3,t-1}+ \\ \Phi_{1,31}\Delta CNH_{t-1} + \Phi_{1,32}\Delta CNY_{t-1} + \Phi_{1,33}\Delta HE_{t-1} + \Phi_{1,34}\Delta YE_{t-1}+ \\ \Phi_{2,31}\Delta CNH_{t-2} + \Phi_{2,32}\Delta CNY_{t-2} + \Phi_{2,33}\Delta HE_{t-2} + \Phi_{2,34}\Delta YE_{t-2}+ \\ \Phi_{3,31}\Delta CNH_{t-3} + \Phi_{3,32}\Delta CNY_{t-3} + \Phi_{3,33}\Delta HE_{t-3} + \Phi_{3,34}\Delta YE_{t-3}+ \\ \beta_{31}\Delta EI_t + \beta_{32}\Delta FX_t + \epsilon_{3t}\end{aligned} \tag{2.59}$$

$$\begin{aligned}\Delta YE_t = \psi_4 + \Pi_{41}EC_{1,t-1} + \Pi_{42}EC_{2,t-1} + \Pi_{43}EC_{3,t-1}+ \\ \Phi_{1,41}\Delta CNH_{t-1} + \Phi_{1,42}\Delta CNY_{t-1} + \Phi_{1,43}\Delta HE_{t-1} + \Phi_{1,44}\Delta YE_{t-1}+ \\ \Phi_{2,41}\Delta CNH_{t-2} + \Phi_{2,42}\Delta CNY_{t-2} + \Phi_{2,43}\Delta HE_{t-2} + \Phi_{2,44}\Delta YE_{t-2}+ \\ \Phi_{3,41}\Delta CNH_{t-3} + \Phi_{3,42}\Delta CNY_{t-3} + \Phi_{3,43}\Delta HE_{t-3} + \Phi_{3,44}\Delta YE_{t-3}+ \\ \beta_{41}\Delta EI_t + \beta_{42}\Delta FX_t + \epsilon_{4t}\end{aligned} \tag{2.60}$$

The estimation results are summarized in Tables 2.7 and 2.8 below. In the second regime, the error correction items of the return of onshore exchange rate are statistically significant while that of the return of offshore exchange rate are not. Compared with the result in Table 2.4, the onshore exchange rate becomes more sensitive to the long-run equilibrium, but the offshore exchange rate deviates from the equilibrium relationship persistently.

The information of short-term correlation between the two exchange rates and economic variables is also shown in Table 2.7 and 2.8. Column (4-1) in the tables shows that the second order lagged return of onshore exchange rate and the third order lagged return of offshore exchange rates have significant negative marginal effects (-0.077, -0.187) on the return of offshore exchange rate. In addition, the change of foreign reserves has significant positive (0.603) impact on the return of offshore exchange rate. Compared with column (2-1) in Table 2.4, the major determinants of offshore exchange rate have changed. The marginal effects from the previous offshore exchange rate and change of foreign reserves to the return of offshore exchange rate have increased. It implies that the offshore market participants focus on the adjustment in PBoC's foreign reserves, which represents the operation of intervention.

Column (4-2) in the Tables 2.7 and 2.8 suggests that both the first and second order lagged return of onshore exchange rate have negative impacts (-0.208, -0.237) on the return of onshore exchange rate. In addition, the first order lagged offshore return rate influences the return of onshore exchange rate positively (0.221). Besides this, both return of the effective exchange rate and the change of foreign exchanges have positive (0.081, 0.158) impacts on the return of onshore exchange rate. Compared with column (2-2) in Table 2.4, the influence from the onshore inter-bank forward rate and the effective exchange rate have declined significantly after the reform. Meanwhile, the significant coefficient of foreign reserves confirms the existence of intervention by the PBoC.

In Column (4-3) of the tables, the first order lagged returns of offshore exchange rate has positive (0.324) influence on the return of offshore market forward rate. Additionally, both the first and third order lagged returns of onshore exchange rate have positive (0.168, 0.127) impacts on the return of offshore market forward rate.

Moreover, the first order lagged return of offshore forward rate has a negative (-0.352) impact, indicating the auto-correlation effect of the offshore forward rate. Furthermore, the change of the foreign reserves also has a positive (0.369) influence, therefore the change of foreign reserves not only influences the market participants' investing decision, but also influences their expectations.

Column (4-4) shows that both the first and second lagged order return of onshore exchange rate have positive (0.202, 0.233) influence on the return of onshore inter-bank forward rate. Besides, the first order lagged return of offshore forward rate has a significant negative (-0.156) impact on the return of onshore inter-bank forward rate. Compared with Column (2-4) in Table 2.4, the influence from both the onshore and offshore returns of exchange rate decreases.

Moreover, by comparing Table 2.8 and 2.6, the adjusted R^2 of offshore case increases from 0.02 to 0.16, and that of CNY case increases from 0.09 to 0.20. The additional variables, especially the foreign reserves, provide a more powerful explanation on the evolution of the two exchange rates after the reform.

Furthermore, Granger Causality test is also applied to investigate the causal relationships between the returns of two two exchange rates and that of other economic variables in the second stage. The results are shown in Table 2.9.

The results in Table 2.9 reject the null hypothesis that no causality relationship exists between the realized returns of offshore and onshore exchange rates. The offshore exchange rate is Granger cause of the onshore exchange rate, however, the onshore exchange rate is not Granger cause of the offshore exchange rate. In addition, the offshore exchange rate is uni-directional Granger cause to both the onshore and offshore forward rates. These results indicate that the offshore exchange rate is dominant in both the expectation of onshore and offshore market agents. Moreover, the change of foreign reserves is uni-directional Granger cause of the offshore exchange rate return. Besides this, the return of onshore exchange rate is also Granger cause to both the return of onshore and offshore forward rates. The change of effective exchange rate index and that of foreign reserves are

Granger cause of the return of onshore exchange rate, although the significance levels are not high. Overall, the causality test results are consistent with the estimation results.

Compared with the Granger Causality test results in Table 2.5, the results in Table 2.9 show that the onshore exchange rate, the offshore forward rate and the onshore inter-bank forward rate are no longer significant determinants of the offshore exchange rate after the exchange rate reform. However, the foreign reserves becomes a significant factor to determine the offshore exchange rate. In addition, the offshore forward rate is not a significant factor of the onshore exchange rate determination. Therefore, the role of leader has shifted from the onshore market to the offshore market. Moreover, the foreign reserves become a significant factor to determine the onshore exchange rate like the offshore exchange rate in the second regime. It indicates that the PBoC's intervention impacts both the onshore and offshore markets.

2.6 Calibration

In this section, we will perform calibration on both the onshore and offshore exchange rate paths in the theoretical model, in order to investigate whether they can describe the evolution of historical RMB exchange rates.

The exogenous parameters in the theoretical model need to be determined. In equation (2.14), θ is used to balance two conflicting motivations of central bank in daily open market operations, of which one is to smooth the exchange rate fluctuation and the other one is to maintain the wealth of foreign reserve. Since the PBoC is a heavy interventionist, θ is set as 0.015, which indicates that the primary goal of the PBoC is to manage the onshore exchange rate within a specific range instead of speculating in the market.

Parameter Φ in equation (2.20) is determined empirically. The data of BIS RMB Effective Exchange Rate Index, which has been used in empirical analysis of Section 5, is applied to equation (2.20) to estimate parameter Φ. The result equals to 0.002.

Additionally, parameters ρ_1, ρ_2, ρ_3 in equation (2.9), which describes the dynamic path of offshore exchange rate, follow the parameter settings in [48] and [35]. Therefore, $\lambda, \mu, \alpha, \eta, \pi^F, \pi^C$ are set to as $0.100, 0.150, 0.600, 0.100, 0.200, 0.600$ respectively. By substituting the series of values in equation (2.9), the parameters $\rho_0, \rho_1, \rho_2, \rho_3$, are set as 0.122, -0.073, -0.049 and 0.878, respectively. Moreover, δ in equation (2.11) is set as 1, referring to [40]. γ and χ, which are the weights in value function of onshore market participants, are set as 0.700 and 0.500. By substituting values of δ, γ, χ in equation (2.13), the parameters κ_1, κ_2, κ_3, κ_4 equal to -0.700, -0.500, -0.200 and 0.400 respectively. These parameters are substituted in the equation system (2.32) - (2.36) in order to solve the parameters τ, ξ, φ, Ω and ν.

$$\tau = 0.002,\ \xi = 0.946,\ \varphi = 0.006,\ \Omega = 0.002,\ \nu = -0.014 \tag{2.61}$$

The dynamic paths of the offshore and onshore exchange rates are expressed as:

$$\tilde{e}_t = 0.122 - 0.073e_t - 0.049\bar{e}_t + 0.878\tilde{e}_{t-1} \tag{2.62}$$

$$e_t = 0.002 + 0.946e_{t-1} + 0.006\bar{e}_t + 0.002\overline{FX}_{t-1} - 0.014\tilde{e}_{t-1} \tag{2.63}$$

Since the offshore exchange rate $\tilde{e}_t$ is determined after the PBoC issues the onshore exchange rate e_t, then through substituting (2.63) into (2.62), the path of the offshore exchange rate can be rewritten as:

$$\tilde{e}_t = 0.122 + 0.879\tilde{e}_{t-1} - 0.054\bar{e}_t - 0.069e_{t-1} - 0.002\overline{FX}_{t-1} \tag{2.64}$$

where the items with coefficients less than 0.001 are ignored. It is necessary to compare the paths generated from the theoretical model with the results from the empirical analysis. By extracting the estimated significant coefficients from Table 2.4, the paths of offshore and onshore exchange rates in the first regime can be written as follows. It is notable that the error correction items $EC_{1,t-1}$ and $EC_{2,t-1}$ have been replaced by the estimated co-integrated series.

$$\begin{aligned} CNH_t = -0.006 + 0.824CNH_{t-1} + 0.372CNY_{t-1} + 0.103HE_{t-1} + \\ 0.009YE_{t-1} - 0.257CNY_{t-2} - 0.110HE_{t-2} - 0.065YE_{t-2} \end{aligned} \tag{2.65}$$

$$\begin{aligned} CNY_t = -0.002 + 0.161EI_t + 0.241CNH_{t-1} + 0.802CNY_{t-1} + \\ 0.002HE_{t-1} + 0.020YE_{t-1} - 0.161EI_{t-1} - 0.128CNH_{t-2} - \\ 0.103CNY_{t-2} + 0.093YE_{t-2} - 0.082CNH_{t-3} + 0.244CNY_{t-3} \end{aligned} \tag{2.66}$$

Similarly, by extracting the estimated significant coefficients from Tables 2.7 and 2.8, the paths of offshore and onshore exchange rates in the second regime can be written as:

$$\begin{aligned} CNH_t = 0.603FX_t + 0.998CNH_{t-1} - 0.042CNY_{t-1} + 0.043HE_{t-1} + \\ 0.002YE_{t-1} - 0.603FX_{t-1} - 0.077CNH_{t-2} - 0.110CNH_{t-3} + \\ 0.187CNH_{t-4} \end{aligned} \tag{2.67}$$

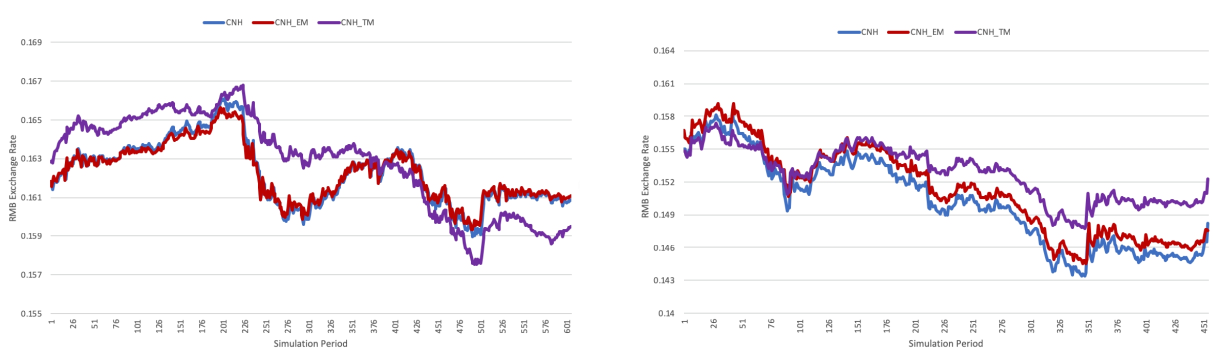
$$\begin{aligned} CNY_t = 0.003 + 0.081EI_t + 0.158FX_t + 0.204CNH_{t-1} + 0.749CNY_{t-1} - \\ 0.071HE_{t-1} + 0.028YE_{t-1} - 0.081EI_{t-1} - 0.158FX_{t-1} - \\ 0.147CNH_{t-2} - 0.038CNY_{t-2} + 0.175CNY_{t-3} \end{aligned} \tag{2.68}$$

By comparing equations (2.64), (2.65) and (2.67) on the path of offshore exchange rate, it is found that the marginal effects from first order lagged offshore exchange rate on itself in equation (2.64) and (2.65) are quite similar (0.879, 0.824). Meanwhile, the marginal impact of the first order lagged offshore exchange rate in equation (2.67) is close to one (0.998). Additionally, the marginal effect from the effective exchange rate is negative (-0.054) but close to zero in the simulated path (2.64). However, the influence of effective exchange rate is not significant in both (2.65) and (2.67), which implies that the offshore market agents are not concerned about the effective exchange rate as the theoretical model describes. Moreover, the marginal effect from previous onshore exchange rate is negative (-0.069) in the theoretical path. On the other hand, both the first and second order lagged onshore exchange rate have impacts on the offshore exchange rate in the first stage (0.372, -0.257). Furthermore, in the second stage, both the second and third lagged offshore change rates have impacts on the spot offshore exchange rate (-0.077, 0.110), and even the fourth order lagged offshore exchange rate has a positive (0.187) impact. Therefore, the duration of influence from the previous exchange rates on the current offshore exchange rate is longer than what the theoretical model suggests.

Similarly, by comparing equations (2.63), (2.66) and (2.68) of onshore exchange rate, the influence from first order lagged onshore exchange rate on the series itself is strongly positive (0.946) in the theoretical path, while the corresponding impacts of first order lagged onshore exchange rate on the series are smaller (0.802, 0.749) in both stages of the empirical equations. In the empirical equations, the second and third order lagged items also have impacts (-0.103, 0.244; -0.038, 0.175), which indicates that the evolution of onshore exchange rate depends on its previous path. In addition, the effective exchange rate has a positive (0.006) effect on the onshore exchange rate in the theoretical equation. The empirical results suggest that the change of effective exchange rate impacts the onshore exchange rate positively (0.161, 0.081) in both stages. Moreover, the influence of first order lagged offshore exchange rate is negative (-0.014) in the theoretical equation and positive in two empirical equations (0.241, 0.204). However, the empirical results show that the second order lagged offshore exchange rates performs negative (-0.128, -0.147) influence on the onshore exchange rate in the both stages.

Furthermore, the weight of offshore forward rate in determining onshore exchange rate increases from the first stage to the second stage. In the first stage, only the first order lagged offshore forward rate has a small positive (0.002) impact on the onshore exchange rate. However, in the second stage, the marginal effect of offshore forward rate increases to 0.071. The offshore market expectation becomes more significant in the determination of onshore exchange rate.

The two plots of Figure 2.8 show the simulated paths of offshore exchange rates through theoretical equation (2.64) and empirical equations (2.65), (2.67) in each stage respectively. In Figure 2.8, the purple series, which is generated by the theoretical equation (2.64), moves in a similar pattern as that of historical offshore exchange rate. The red series, which is generated from the empirical equations (2.65) and (2.67), overlap with the blue series of the historical offshore series.

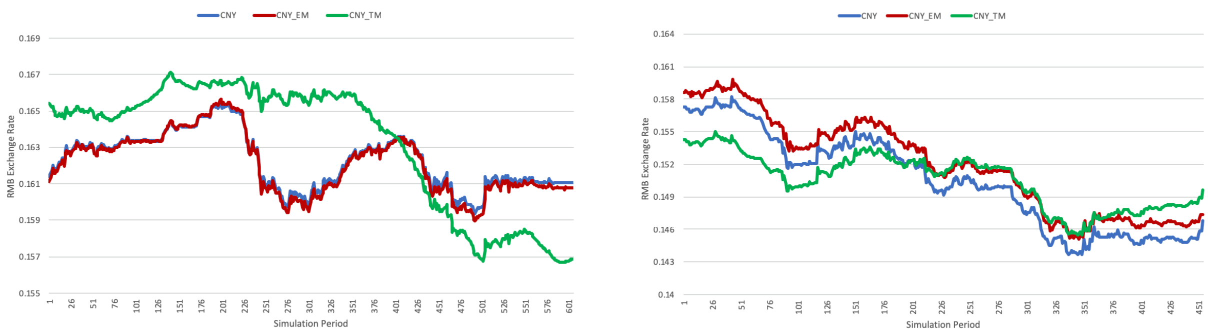


(a) Regime 1

(b) Regime 2

Figure 2.8 Simulation of the Offshore Exchange Rate

Correspondingly, the two plots of Figure 2.9 show the simulated paths of onshore exchange rates through theoretical equation (2.63) and empirical equations (2.66), (2.68) in each stage respectively. The red series, which is generated from the empirical equations, matches with the historical onshore exchange rate. However, the green series, which is generated by theoretical equation (2.63), displays a certain level of deviation from the realized onshore exchange rate, especially at the time point close to the reform of exchange in the first stage.



(a) Regime 1   (b) Regime 2

Figure 2.9 Simulation of the Onshore Exchange Rate

Overall, the main components of theoretical and empirical equations are the same, although the market expectations are expressed explicitly in the empirical equations. Compared with the theoretical equations, more influences of the lagging items are incorporated to the empirical equations which indicates that the previous pricing of exchange rates has longer impact on the decision of agents in both onshore and offshore markets. Besides this, not the effective exchange rate index but the foreign reserve plays an key role in offshore exchange rate determination. Additionally, after "811 Reform of RMB Exchange Rate", the influence of offshore market including both spot and forward markets on the onshore market increased, while the influence of onshore market on the offshore market declined. It indicates that the leading role between the two exchange rates has been shifted from the onshore market to the offshore market.

2.7 Conclusion

With the internationalization of RMB, there is a boost in the prosperity of the offshore exchange market. In recent years, the interest of market participants, policy makers and scholars has been attracted by the offshore exchange rate. Many of the literature focuses on investigating the dynamic correlation between the onshore and offshore RMB exchange rates, while an illustration of the mechanism, in which how the two exchange rates are determined and impact each other, has been lacking. This paper develops a theoretical model of pricing RMB exchange rate, in order to address the gap in market micro-structure theory applied in RMB exchange rate. Specifically, the model describes the mechanism of two RMB exchange rates of which one is determined in a free-floating exchange rate system and the other one is determined in a managed-floating exchange rate system with central bank's intervention simultaneously. In the offshore market, the market participants make their trading decisions with reference to the onshore exchange rate. In the onshore market, the commercial banks maximize their transaction value functions including the effective exchange rate, the onshore and offshore exchange rates. The PBoC determines the official onshore exchange rate taking the reaction function of commercial banks into account.

VECM with exogenous variables is applied, in order to capture the behavior of the two exchange rates. The onshore and offshore exchange rates display different long-term and short-term interactions before and after "811 Reform of RMB Exchange Rate". The empirical results show that before the reform, the offshore exchange rate, rather than the onshore exchange rate, adjusts to the estimated long-term relationships. Meanwhile, after the reform, the onshore exchange rate, rather than the offshore exchange rate, is sensitive to the estimated long-term relationships. In addition, in the first regime, the onshore exchange rate has a greater impact on the offshore exchange rate, while in the second regime, the offshore exchange rate has a greater impact on the onshore exchange rate.

Moreover, the short-term causal relationship between the spot rates and forward rates vary over time and the foreign exchange reserves have a significant impact on both exchange rates after the exchange rate reform. Besides this, the paths of onshore and offshore exchange rate simulated by the theoretical model have similar features to the paths generated from the estimation. However, the empirical results confirm that not only the contemporaneous effect but also the lagged effect exists, which is not reflected in the theoretical model. Additionally, the effective exchange rate has no significant impact on the offshore market exchange rate as the theoretical model suggests.

This study reveals the mechanism on how the onshore exchange rate interacts with the offshore exchange rate. However, there are still some limitations. For example, the volatility of the exchange rate has not been reflected in the dynamic equations. In general, the second order moment of price will have an impact on the decision of market agents. Besides this, the central parity set by the PBoC restricts the fluctuation range of exchange rate which can be applied to refine the model. These limitations lead to promising research in future.

Table 2.4 Estimation of Multi-Variate VECM with Exogenous Variables Feb 2013—Aug 2015

Parameters	VECM(3)			
	(2-1)	(2-2)	(2-3)	(2-4)
	ΔCNH_t	ΔCNY_t	ΔHE_t	ΔYE_t
$Constant$	-0.006	-0.002	0.001	-0.002
	(0.001)***	(0.001)*	(0.001)	(0.001)
$EC_{1,t-1}$	-0.108	0.033	-0.031	0.182
	(0.032)***	(0.027)	(0.021)	(0.025)***
$EC_{2,t-1}$	-0.068	0.031	-0.002	-0.050
	(0.021)**	(0.020)*	(0.015)	(0.019)**
ΔCNH_{t-1}	0.001	0.210	0.296	-0.021
	(0.053)	(0.047)***	(0.036)***	(0.044)
ΔCNY_{t-1}	0.257	-0.143	-0.112	0.301
	(0.063)***	(0.056)*	(0.043)**	(0.052)***
ΔHE_{t-1}	0.110	0.067	-0.255	0.046
	(0.045)*	(0.055)	(0.043)***	(0.052)
ΔYE_{t-1}	-0.065	0.053	0.031	-0.211
	(0.020)*	(0.042)	(0.033)***	(0.040)***
ΔCNH_{t-2}	-0.033	0.082	0.077	-0.047
	(0.053)	(0.044)*	(0.035)*	(0.042)
ΔCNY_{t-2}	-0.082	-0.244	-0.097	0.250
	(0.065)	(0.056)***	(0.043)**	(0.052)***
ΔHE_{t-2}	0.031	0.082	-0.088	0.043
	(0.063)	(0.053)	(0.041)*	(0.050)
ΔYE_{t-2}	0.033	0.093	0.007	-0.050
	(0.044)	(0.037)*	(0.029)	(0.035)
ΔEI_t	0.013	0.161	0.025	0.012
	(0.087)	(0.074)*	(0.058)	(0.069)
ΔFX_t	0.116	0.030	0.049	0.029
	(0.071)	(0.060)	(0.047)	(0.056)
$Adj.R^2$	0.18	0.17	0.22	0.26

Note: Robust standard errors in parentheses *** p <0.01, ** p <0.05, * p <0.1.

Table 2.5 The Granger Gausality Test for Multi-Variate VECM with Exogenous Variables Feb 2013—Aug 2015

Null Hypothesis	$F-$Statistic	$Pr(> F)$
$\Delta CNH_t \neq> \Delta CNY_t$	11.207	< 0.001***
$\Delta CNY_t \neq> \Delta CNH_t$	8.211	< 0.001***
$\Delta CNH_t \neq> \Delta HE_t$	14.552	< 0.001***
$\Delta HE_t \neq> \Delta CNH_t$	3.198	0.007**
$\Delta CNH_t \neq> \Delta YE_t$	1.264	0.278
$\Delta YE_t \neq> \Delta CNH_t$	29.287	< 0.001***
$\Delta CNH_t \neq> \Delta EI_t$	0.350	0.883
$\Delta EI_t \neq> \Delta CNH_t$	1.717	0.129
$\Delta CNH_t \neq> \Delta FX_t$	0.515	0.769
$\Delta FX_t \neq> \Delta CNH_t$	0.404	0.850
$\Delta CNY_t \neq> \Delta HE_t$	1.126	0.315
$\Delta HE_t \neq> \Delta CNY_t$	2.874	0.014*
$\Delta CNY_t \neq> \Delta YE_t$	50.621	< 0.001***
$\Delta YE_t \neq> \Delta CNY_t$	0.977	0.431
$\Delta CNY_t \neq> \Delta EI_t$	0.536	0.749
$\Delta EI_t \neq> \Delta CNY_t$	2.100	0.064*
$\Delta CNY_t \neq> \Delta FX_t$	0.237	0.946
$\Delta FX_t \neq> \Delta CNY_t$	0.058	0.998

Table 2.6 Estimation of Bi-Variate CNH_t & CNY_t VECM Aug 2015—May 2017

Parameters	VECM(4)	
	(3-1)	(3-2)
	ΔCNH_t	ΔCNY_t
$Constant$	-0.003	0.007
	(<0.001)	(<0.001)
EC_{t-1}	-0.050	0.096
	(0.037)	(0.024)**
ΔCNH_{t-1}	0.064	0.152
	(0.060)	(0.039)***
ΔCNY_{t-1}	-0.006	-0.161
	(0.085)	(0.055)***
ΔCNH_{t-2}	0.034	0.022
	(0.060)	(0.039)
ΔCNY_{t-2}	-0.135	-0.180
	(0.084)*	(0.054)***
ΔCNH_{t-3}	-0.135	-0.011
	(0.059)	(0.038)
ΔCNY_{t-3}	0.083	0.070
	(0.082)	(0.053)
$Adj.R^2$	0.02	0.09

Note: Robust standard errors in parentheses *** p <0.01, ** p <0.05, * p <0.1.

Table 2.7 Estimation of Multi-Variate VECM with Exogenous Variables Aug 2015—May 2017 A

Parameters	VECM(4)			
	(4-1)	(4-2)	(4-3)	(4-4)
	ΔCNH_t	ΔCNY_t	ΔHE_t	ΔYE_t
$Constant$	0.002	0.003	0.002	0.005
	(0.002)	(0.001)**	(0.002)	(0.001)***
$EC_{1,t-1}$	0.012	-0.003	0.005	-0.028
	(0.007)	(0.005)*	(0.005)	(0.001)***
$EC_{2,t-1}$	0.027	0.112	0.044	0.072
	(0.037)	(0.025)**	(0.024)*	(0.020)**
$EC_{3,t-1}$	-0.002	-0.002	0.001	0.001
	(0.000)*	(0.001)*	(0.001)	(0.001)
ΔCNH_{t-1}	0.048	0.221	0.324	0.024
	(0.097)	(0.049)**	(2e-16)***	(0.039)
ΔCNY_{t-1}	-0.004	-0.208	0.168	0.202
	(0.063)	(0.065)*	(0.006)**	(0.054)***
ΔHE_{t-1}	0.041	-0.063	-0.352	-0.156
	(0.108)	(0.075)	(0.069)***	(0.060)*
ΔYE_{t-1}	-0.188	0.003	-0.050	-0.005
	(0.106)	(0.058)	(0.055)	(0.046)

Note: Robust standard errors in parentheses *** p <0.01, ** p <0.05, * p <0.1.

Table 2.8 Estimation of Multi-Variate VECM with Exogenous Variables Aug 2015—May 2017 B

Parameters	VECM(4)			
	(4-1)	(4-2)	(4-3)	(4-4)
	ΔCNH_t	ΔCNY_t	ΔHE_t	ΔYE_t
ΔCNH_{t-2}	-0.077	0.062	0.010	-0.013
	(0.010)**	(0.047)	(0.045)	(0.039)
ΔCNY_{t-2}	-0.023	-0.237	0.046	0.233
	(0.075)	(0.067)**	(0.064)	(0.056)***
ΔHE_{t-2}	0.092	0.026	-0.011	-0.158
	(0.111)	(0.074)	(0.078)	(0.062)
ΔYE_{t-2}	-0.092	0.049	-0.010	-0.005
	(0.083)	(0.056)	(0.053)	(0.046)
ΔCNH_{t-3}	-0.187	-0.006	-0.020	0.010
	(0.067)**	(0.045)	(0.042)	(0.037)
ΔCNY_{t-3}	0.135	0.019	0.127	-0.027
	(0.107)	(0.067)	(0.043)**	(0.055)
ΔHE_{t-3}	-0.004	0.033	-0.003	-0.009
	(0.099)	(0.065)	(0.062)	(0.054)
ΔYE_{t-3}	0.035	0.015	-0.039	-0.003
	(0.078)	(0.052)	(0.050)	(0.043)
ΔEI_t	-0.113	0.081	-0.151	-0.017
	(0.417)	(0.028)*	(0.264)	(0.231)
ΔFX_t	0.603	0.158	0.369	-0.329
	(0.182)**	(0.093)*	(0.166)*	(0.185)
$Adj.R^2$	0.16	0.20	0.25	0.27

Note: Robust standard errors in parentheses *** $p<0.01$, ** $p<0.05$, * $p<0.1$.

Table 2.9 The Granger Gausality Test for Multi-Variate VECM with Exogenous Variables Aug 2015—May 2017

Null Hypothesis	$F-$Statistic	$Pr(> F)$
$\Delta CNH_t \neq> \Delta CNY_t$	7.468	< 0.001***
$\Delta CNY_t \neq> \Delta CNH_t$	0.699	0.625
$\Delta CNH_t \neq> \Delta HE_t$	24.894	< 0.001***
$\Delta HE_t \neq> \Delta CNH_t$	0.564	0.727
$\Delta CNH_t \neq> \Delta YE_t$	9.865	< 0.001***
$\Delta YE_t \neq> \Delta CNH_t$	1.040	0.394
$\Delta CNH_t \neq> \Delta EI_t$	0.528	0.755
$\Delta EI_t \neq> \Delta CNH_t$	0.424	0.832
$\Delta CNH_t \neq> \Delta FX_t$	0.794	0.554
$\Delta FX_t \neq> \Delta CNH_t$	3.140	0.008**
$\Delta CNY_t \neq> \Delta HE_t$	5.596	< 0.001***
$\Delta HE_t \neq> \Delta CNY_t$	1.694	0.135
$\Delta CNY_t \neq> \Delta YE_t$	20.091	< 0.001***
$\Delta YE_t \neq> \Delta CNY_t$	0.623	0.682
$\Delta CNY_t \neq> \Delta EI_t$	0.983	0.427
$\Delta EI_t \neq> \Delta CNY_t$	1.929	0.087*
$\Delta CNY_t \neq> \Delta FX_t$	0.779	0.565
$\Delta FX_t \neq> \Delta CNY_t$	2.035	0.072**

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CHAPTER 3. BITCOIN: A NEW ERA, SCAM OR ANOTHER BUBBLE?

To be submitted

Abstract

This article illustrates the mechanism of Bitcoin market price determination. The previous literature on Bitcoin price mainly focus on the statistical characteristics, especially its high volatility. Following the classical financial market bubble research, this paper develops a model to describe the speculative behaviour in Bitcoin trading market. There are two innovations of this paper. While the previous research on bubble considers the fundamental value as exogenous, the intrinsic value of Bitcoin system grows with probabilities in this study. Besides, the sentiment of speculators is constructed as a quadratic function of the market price. The empirical results suggest the properties of Bitcoin market prices are captured well by the theoretical model. Bitcoin system has some inherent value and can not simply be characterized as a scam. However, the mysterious and innovative feature of Bitcoin system incite the speculative behaviour. The bubble booms and bursts eventually. From this perspective, Bitcoin does not seem to be much different from those famous bubble events in the financial history.

Keywords: Bitcoin system, Intrinsic value, Market bubble, Fundamentalist, Speculator, Autoregressive Conditional Heteroskedasticity

3.1 Introduction

Cryptic-currency is a digital asset designed as an internet-based exchange medium with a decentralized consensus to manage the issuing of cryptic-currencies and to verify transactions. Bitcoin system is the first cryptic-currency system proposed by Nakamoto in 2008. In this system, Bitcoin network, Bitcoin and Bitcoin block chain are three constituents. Bitcoin miners and users are two major participants.

The economic research on Bitcoin system can be divided into two categories: The first category focuses on the behavioural analysis of Bitcoin miners, particularly on how they respond to the potential profits of the mining industry. The second one studies which factors determine the market price of Bitcoin measured by fiat money, if it is considered as a type of digital asset. In this paper, we focus on the second issue, and the first issue will be discussed in another paper separately.

Recall the history of Bitcoin, the geeks participated in Bitcoin mining out of curiosity. The winner of the mining competition can only exchange Bitcoins within a small community-"Bitcoin community", and the price of Bitcoin was determined freely and discontinuously. In other words, different prices may arise in simultaneous transactions of Bitcoins redeeming to fiat currencies. This situation has been changed by the emergence of Bitcoin trading platforms, and global uniform price of Bitcoin was formed gradually. These Bitcoin trading platforms (Coinbase, Bitfinex, Bitstamp, etc) have some functions similar to that of traditional foreign exchange markets and stock exchange markets. They offer market making service to match agents' transactions (Bidding & Asking). More importantly, speculation can be performed by the investors on Bitcoin in the exchange market.

The historical Bitcoin price is displayed in Figure 3.1, of which the time horizon is from July 16^{th}, 2010 to May 31^{st}, 2018. Before 2013, the price of Bitcoin had always been less than \$20. The first sharp rise of Bitcoin price appeared at the end of 2013. The price exceeded \$1,000 for the first time. After surpassing \$1,000, the price declined persistently till mid 2015. The lowest price during the two years was around \$100. From August 2015, the price of Bitcoin ignited second round of rise and reached the peak of \$5,000 at the end of August, 2017. After a very short term of reversion, the

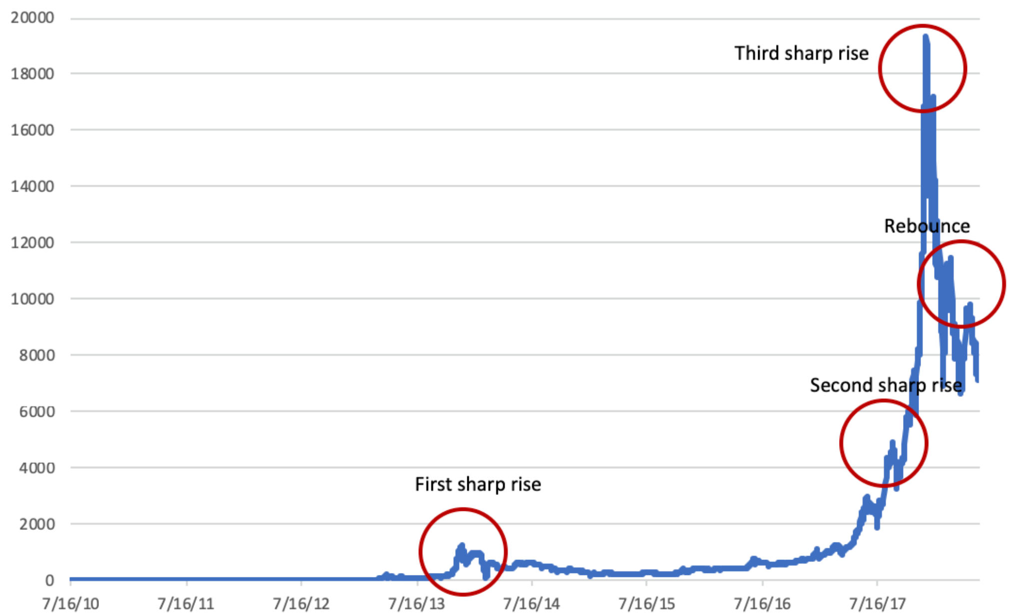


Figure 3.1 Historical Bitcoin Market Price: July 2010-May 2018

price rushed to $20,000 and then dropped dramatically to $6,000. Although re-bounce appeared once in a while, the price kept declining in the long run.

Bitcoin system is viewed as an innovative un-retroactive anonymous payment system. However, the high volatility and rocket-like soaring in the price of Bitcoin imply that Bitcoin system is more likely to be a speculative tool rather than a regular payment system. Following the literature in financial behaviour analysis, a dynamic model of Bitcoin pricing is developed in this paper. The agents in Bitcoin exchange market construct an investment portfolio of two assets to maximize their utilities, in which one is risky asset which represents Bitcoin, and the other one is risk-free asset. In addition, the market agents are divided in two types: fundamentalists and speculators. These two types of investors apply their individual pricing mechanisms to determine their optimal allocations of Bitcoin.

The fundamentalists have the ability to collect information about Bitcoin system. Particularly, the intrinsic value of Bitcoin system plays a key role in their Bitcoin price determination. Since

Bitcoin system is innovative, the valuation of Bitcoin system has not come to a consensus. The fundamentalists have to generate their opinions about Bitcoin system and predict the price based on their own methods. On the other side, the speculators in the Bitcoin exchange market are unable or not keen to obtain the information on intrinsic value of Bitcoin system. They apply the heuristic methods, including trend following and social imitation, to determine their investment strategies. The interaction between these two types of investors determines the market price of Bitcoin at each period.

The statistical analysis on both historical price series and simulated price series generated by the theoretical model is performed. The results show that the model of Bitcoin market price reproduces some classical characteristics of speculative behaviour in financial market. In particular, the feature of fat tail has been detected. Significant ARCH effects (volatility clustering) are observed after filtering out long-term trends. Additionally, the results also suggest that the spot price of Bitcoin is closely related to its previous price, which implies the price is sensitive to its historical information. As designed in the theoretical model, the speculators refer to previous Bitcoin prices to determine their investment strategies.

The remainder of this paper is organized as follows. A review of related literature is provided in Section 3.2. In Section 3.3, the valuation of Bitcoin system is discussed. A theoretical model of Bitcoin market pricing mechanism is developed in Section 3.4. In Section 3.5, the empirical analysis is performed on the historical Bitcoin price series and that simulated from the theoretical model. Finally, the conclusion is made in Section 3.6.

3.2 Literature Review

This paper is aimed to contribute to the economic literature on cryptic-currency and related block chain technologies, particularly with a focus on Bitcoin market price determination which serves as an exchange medium in Bitcoin system. [1] introduced the anonymization mechanism of transaction in Bitcoin network. There is a series of literature ([2], [3], [4], [5], [6], [7], [8], [9], [10]) that investigated the efficiency and security of Bitcoin system. The majority of research suggested that anonymization of Bitcoin network is partially effective. Avoiding 51% of global Hash power from aggregating can not guarantee that the block chain will not be tampered. Bitcoin protocol is distorted by mining pool organization and false information may spread in Bitcoin network. Additionally, potential tracking users' information and anti-anonymization attacks have also been detected.

Correspondingly, a series of literature proposed new technologies and solutions aimed at enhancing privacy and anonymity of Bitcoin system without disrupting the protocol. These new technologies can be broadly classified into three categories: peer-to-peer mixing protocols ([11], [12], [13]); distributed mixing networks ([14], [15]) and Bitcoin extensions or Altcoins ([16], [17]). Therefore, Bitcoin system is still in the process of development.

Potential advantages of cryptic-currencies over the traditional payment systems and the applications of block chain technology in future have been studied. [18] illustrated that block chain technology may reduce the transaction costs caused by information asymmetry between purchasers and sellers. [19] proposed that block chain technology, which provides a decentralized consensus, reshapes the industry organization and market structure. The smart contracts can ease information asymmetry and improve welfare by providing easier entry-exit and promoting the competition.

Moreover, [20] and [21] performed case studies to illustrate how block chain technology creates spontaneous organizations which may contribute to new types of economic organization and governance. However, it is notable that most of the block chain applications described in these literature are still at early trial stage, and are not yet ready for large-scale applications.

Some other literature investigated the Bitcoin **usage** realized in the world. Usage means Bitcoin works as an exchange medium in Bitcoin system instead of being traded as an asset in exchange market. [22] found that in-retroactive feature of Bitcoin system increases the probability of fraudulence, compared with the traditional payment systems. [23] claimed that purchasing drugs and other illegal products is the largest-scale application of Bitcoin usage. Moreover, [24] surveyed vendors which have accepted Bitcoin for online transactions. They claimed that Bitcoin plays a role in circumventing restrictions from government.

Valuation of products based on internet is to some extent of controversy, particularly for Bitcoin system. Till now, one frequently referred valuation formula for network and digital products is Metcalf's Law which is proposed by [25]. Value of network is proportional to the square of node amount which increases with the number of users. Some literature attempted to validate Metcalfe's law on large-scale social network, such as Facebook ([26], [27]) and Tencent, the largest social network in China ([28]). The results show that the relationship between value of network and number of its nodes may depend on the specific type of network.

A few literature ([29], [30]) applied Metcalf's Law to evaluate the cryptic-currency system, especially for Bitcoin system. The results showed that the growth of network value is related to the number of unique addresses in the network which works as proxy of active users. Metcalf's law fits the valuation of network well.

Focused on fundamental value of Bitcoin system, [31] proposed that technology development reduces the cost of keeping public record. Digital ledger may become a substitute for physical currency. [32] claimed that Bitcoin has nature of "private currency". "Private currency" is the currency issued by a private group rather than a government. Bitcoin users are agents who transfer their wealth (remittance) without using legal currency. However, transferring wealth by Bitcoin is accompanied by risks caused by the uncertainty of Bitcoin technology. Bitcoin users may lose the wealth they transferred. Therefore, valuation of Bitcoin system is uncertain, which depends on whether it can function as a mature exchange medium. The longer Bitcoin system runs, the stronger public's belief in Bitcoin system will be, and the more people will be willing to use Bitcoin.

Besides this, [1] stated that a fixed number (21,000,000) of Bitcoins are contained in blocks. This artificial scarcity setting is easy to associate with the scarcity of natural precious metal. Characteristics of scarcity and tangibility are combined together in precious metals. Compared with other digital products, Bitcoin belongs to digital currency but can not be replicated infinitely. In each transaction, every Bitcoin only associates with one wallet each time. Additionally, mining competition in Bitcoin network guarantees that only one winner in each round can obtain Bitcoin. This competition results in uneven distribution of Bitcoin. Therefore, Bitcoin can be viewed as a kind of digital precious metals. Some studies ([33], [34], [35], [36] and [37]) compared Bitcoin with traditional precious metal such as gold. These literature investigated whether Bitcoin can hedge risk like precious metals in the financial market. The empirical results suggested weak correlation exists between Bitcoin and other risky assets. They claimed that Bitcoin is a valuable asset for effective diversification and risk hedging.

A series of literature ([38]; [39]; [40]; [41]; [42]; [43]; [44]; [45]; [46]; [47]; [48] and [49]) have researched on statistical features of historical market price of Bitcoin, particularly on the high volatility and predictability. [45] is representative among these literature, which claimed that Bitcoin does not behave like general monetary system according to the common standards used by economists. The rapid appreciation of Bitcoin against fiat currency and the high volatility of market price is a challenge for Bitcoin to serve as a currency in circulation. This exchange rate is not relate with other existing currencies. Therefore, hedging risk through Bitcoin is not reliable.

Moreover, saving wealth in Bitcoin is un-safe because deposit insurance is not provided. Therefore, [45] proposed that Bitcoin is more like a speculative investment rather than a feasible monetary.

The first model of market bubble boom and burst is provided in [50] and [51], which illustrated the mechanism of long-term market price deviation from the fundamentals. The stability of market price is broken by the positive feedback effect from the behaviour of noise traders which is triggered by rational traders' transactions. The positive feedback in turn leads to a short-term positive auto-correlation of return rate. Eventually, rational investors will drag prices back to the fundamentals. This positive feedback effect has been confirmed by a series of empirical studies ([52], [53] and [54])

The study of market agent heterogeneity has been further developed in [55], [56] and [57]. A model of adaptive belief system in financial market is established. Heterogeneous investors with different risk attitudes are adapted to their beliefs according to the previous market prices, their different predictors and expectation functions. In addition, [58] established a financial behavioural model in which herding and contagion effects are incorporated. The fundamentalists trade by observed wrong market price, while noise traders follow the market sentiment. The market price is determined by the interaction between these two types of investors.

Moreover, based on [59], [60] developed a market fraction model which establishes the relationship between stochastic elements and fundamental deterministic forces in the market. This model has the potential to characterize volatility cluster and long-term dependence of asset returns.

Furthermore, a series studies ([61], [62], [63], [64], [65], [66], [67], [68], [69], [59], [70], [71], [72], [73], [74], [75], [76], [77] and [78]) introduced psychological factors in modeling speculators' behaviour, which is driven by the overall evolution of psychology. These models aim at explaining how speculators' behaviour reinforce their predictions. The influence from fluctuations in opinions (bullish & bearish), changes in transaction regimes and long-term memory have been captured in these models. These models suggested that the market agents may be wrong in short term period, but self-fulfilling prophecy will drive them to the fundamental in a long run period.

3.3 Brief Discussion of Market Cap of Bitcoin System

Bitcoin system is an innovative, in-retroactive, non-changeable and anonymous payment system. But it comes with some disadvantages. For example, the inefficiency of decentralized proof-of-work mechanism and limited block capacity lead to transaction congestion. It probably takes one day or a few days for a transaction to be confirmed, in which Bitcoin serves as an exchange medium. Although Bitcoin community has proposed potential schemes to improve efficiency (such as SegWit and Lightning Network), these ideas have not yet been implemented. Therefore, Bitcoin system has never become a main-stream payment instrument like Paypal, and its acceptance rate is much lower than that of main-stream payment system.

Given the number of completed transactions, the market cap of Bitcoin system should not exceed that of widely accepted Paypal, MasterCard or Visa payment system. Figure 3.2 displays the market cap of Bitcoin system and other three major electronic payment companies from July 16^{th}, 2015 to March 29^{th}, 2018. During most of the time horizon, the market cap of Bitcoin system is similar to that of other three companies. However, during the periods from October 2016 to January 2017, and from October 2017 to January 2018, the market cap of Bitcoin system has experienced two surges and plunges. This high volatility and rocket-like soaring in market cap of Bitcoin system can not be explained by the fundamentals, which leads to suspicion that Bitcoin has been applied as a speculative tool rather than a common payment instrument.

Recalling history of Bitcoin development provides a clue of valuation of Bitcoin system. In early stage of Bitcoin development, S. Nakamoto mined out the genesis block and obtained Bitcoin. As

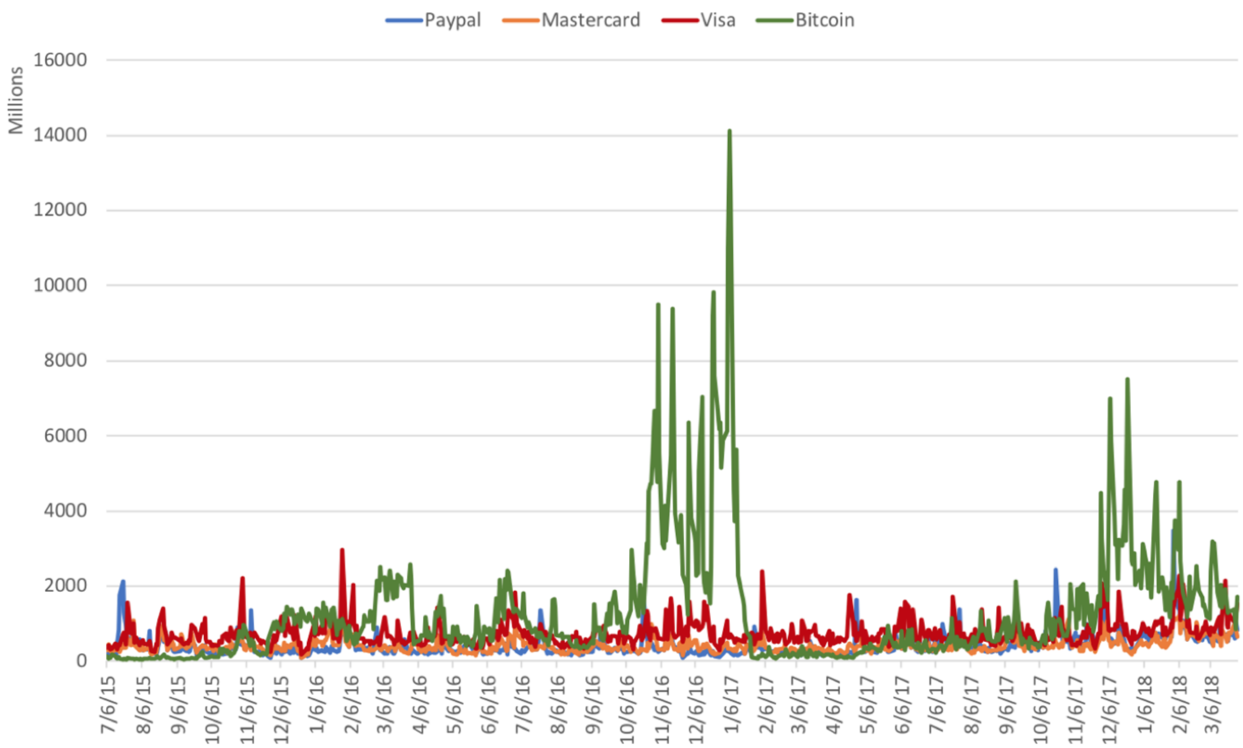


Figure 3.2 Comparison of Market Cap among Payment Systems

known, he transferred these Bitcoins to H. Finny, which is the first transaction recorded in Bitcoin history. Although it is unclear how much H. Finny paid for these Bitcoins, which may be a very small amount of U.S. Dollar, the first price of Bitcoin denominated in U.S. Dollar (fiat currency) was born. Thereafter, more and more geeks participated in mining of blocks because of anarchism or pure fun, and they traded Bitcoins with freely and discontinuously determined price.

The first category of Bitcoin usage related to the real world emerged, when someone discovered that anonymity of Bitcoin system could facilitate money laundering and drug transaction. Agents that perform illegal transactions purchase Bitcoins from the computer geeks. They are willing to accept Bitcoin as a payment instrument because the potential losses caused by unstable price of Bitcoin are worthwhile, compared with the benefit of long time evading from tracking and supervision.

The second category of realized Bitcoin usage comes from the household whose domestic currency system has collapsed, such as Cyprus where banking crisis occurred in 2013, Zimbabwe and Venezuela after 2015. Households in these countries need alternative value storage to reserve their wealth because the domestic currencies became worthless. However, foreign currency is strictly regulated in theses countries and households can not convert their wealth into U.S. dollars or other stable fiat money easily. Bitcoin provides them an alternative choice. Households understand that Bitcoin system is not steady and price is extremely volatile. Nevertheless, Bitcoin may help them recover at least partial losses compared to the possibility that all their wealth may turn into nothing.

Finally, it is undeniable that a few retail stores in Japan, South Korea and Thailand have indeed started accepting Bitcoin as a payment instrument. However, this operation is more similar to a kind of business trial. These stores have never abandoned traditional payment systems.

Overall, using Bitcoin as an exchange medium in the real world mainly comes from agents involved in illegal transactions, households of the countries with collapsed legal currency system and a trail alternative payment supported by a few small business.

Emergence of Bitcoin trading market has changed this situation. Bitcoin exchange market provides a trading platform for the holders of Bitcoin and global uniform price of Bitcoin has been formed gradually. The first Bitcoin trading platform with broad influence in history is MtGox. It was established in 2010, but was declared bankrupt in 2014 due to massive loss of Bitcoin caused by hacking. Currently, large trading platforms include Coinbase, Bitfinex, Bitstamp and so on. These Bitcoin trading platforms have functions similar to that of traditional foreign exchange markets and stock exchange markets. They offer market making service to match individual agents' transactions (Bidding & Asking). More importantly, speculation can be performed by the investors on Bitcoin in the exchange market with unique characteristics. In traditional stock exchange market, traders evaluate price of stock based on the company's performance which reflects fundamental value of the stock. In foreign exchange market, the spot foreign exchange rate is closely linked to effective exchange rate which reflects macro-economic situation. However, measuring the fundamental value of Bitcoin system is controversial.

Innovations always desire high expectation. Anonymity and privacy protection in the process of transferring wealth, potential applications of block chain technology and the mysterious channel of obtaining Bitcoin are attractive. This is very similar to that of so-called "new economy" brought to the public in "Dot-Com" bubble of 1990s. During 1990s, internet, software, computer hardware and telecommunications are the main components of "new economy". The listed companies in these industries are mainly covered in NASDAQ Composite index. The main feature of these stocks is that most of their P/E ratios are higher than 100. In contrast, for companies in so called "old economy" such as Ford and General Motors, their P/E ratios are generally less than 20. The reason for this significant difference is that investors expect the revenues from internet technology will increase more substantially than companies of car sales in future. Therefore investors are more willing to invest in Cisco rather than Ford, even though the former's earnings per share are less than that of the latter.

Market sentiment of Bitcoin trading has experienced similar evolution as that of NASDAQ stock market. Before the crash of NASDAQ index, investors' optimism synchronized with the bullish trend. They generally believe that internet and new technologies can increase market efficiency, stimulate global competition, and create new patterns of business and services. However, these explanations did not hold water after crash.

Similarly, With the rise in price of Bitcoin, many people (media or financial institution) began to discuss the possibility of Bitcoin replacing legal monetary and how block chain technology will change business models. However, after Bitcoin price plummeted, there are fewer and fewer discussions like that.

Finally, the comparison of evolution between NASDAQ index and Bitcoin price is intuitive. After reaching a record high of 5,133 points on March 10^{th}, 2000, NASDAQ composite index dropped to a low point of 3,227 points on April 17^{th}, 2000. Recalling trend of market price of Bitcoin in 2017, it broke through 1,000 U.S. dollars for the first time at the end of January then raise to peak of 19,000 U.S. dollars on December 17^{th} and after that dropped to a low point of 6,992 dollars on February 5^{th}, 2018. The price has experienced a 1,900 % increase and then a drop of more than 63%!

3.4 Theoretical Model

In this section, a model of Bitcoin investors' trading behaviour in the exchange market is developed. The Bitcoin investors are divided into fundamentalists and speculators. The fundamentalists focus on the intrinsic value of Bitcoin system while the speculators explore information from the previous paths of market price. Thus, these two categories of investors apply different rules to predict the market price of Bitcoin and to determine the amount of Bitcoin to purchase or sell. The market price of Bitcoin is determined by the aggregate excess demand of fundamentalists and speculators.

3.4.1 Optimal Amount of Investing in Bitcoin

Following classic literature in financial behaviour analysis ([55], [56], [60] and [79]), the optimal amount of Bitcoin investment is determined in this subsection.

In each period, the investors maximize the expected utility of wealth in the next period by constructing a portfolio, which consists of risky asset (Bitcoin) and risk-free asset, till the next period. For each discrete time $t \in \{1, ..., T\}$, $w_{j,t}$ is defined as exogenous wealth of investor j at the beginning of period t, and $x_{j,t}$ is the amount of wealth that investor j invests in Bitcoin at time t, and the corresponding amount of saving in risk-free asset is $w_{j,t} - x_{j,t}$. Then the wealth of investor j in period $t+1$ is written as:

$$w_{j,t+1} = R_{f,t}(w_{j,t} - x_{j,t}) + \frac{p_{t+1}}{p_t} x_{j,t} \tag{3.1}$$

where $R_{f,t}$ is the return rate on risk-free asset. After rearrangement, wealth in the next period $w_{j,t+1}$ can be written as:

$$w_{j,t+1} = R_{f,t} w_{j,t} + (p_{t+1} - R_{f,t} p_t) z_{j,t} \tag{3.2}$$

here $z_{j,t}=\frac{x_{j,t}}{p_t}$ is the number of Bitcoins purchased by investor j at period t. The first item on the right side of equation (3.2), $R_{f,t} w_{j,t}$, represents exogenous risk-free revenue in the next period, while the second item $(p_{t+1} - R_{f,t} p_t) z_{j,t}$ stands for the change in wealth caused by investment in Bitcoin. Therefore, the effect of potential wealth change is separated out.

For simplicity, the excess return of period $t+1$ is denoted as R_{t+1}, where $R_{t+1} = p_{t+1} - R_{f,t} p_t$. Then conditional mean and variance of return in the next period are:

$$E_{j,t}[w_{j,t+1} - R_{f,t} w_{j,t}] = z_{j,t} E_{j,t}[R_{t+1}] \tag{3.3}$$

$$V_{j,t}[w_{j,t+1} - R_{f,t} w_{j,t}] = z_{j,t}^2 V_{j,t}[R_{t+1}] \tag{3.4}$$

In equations (3.3) and (3.4), conditional mean and variance of wealth in the next period consist of the amount of invested Bitcoins and the expectation of market price.

In addition, the specific form of Bitcoin investor j' s utility function is set as a constant absolute risk aversion (CARA) utility function as follows:

$$U_{j,t} = -e^{-\alpha_{j,t} w_{j,t}} \tag{3.5}$$

where $\alpha_j = A\left(w_{j,t}\right) = -\frac{U''(w_{j,t})}{U'(w_{j,t})}$ is a constant coefficient of absolute risk aversion.

The investor maximizes his or her expected utility with the following objective function:

$$\begin{aligned} Max\ E_t[U_{j,t+1}(w_{j,t+1})] &= -e^{-\alpha_j w_{j,t+1}} \\ &= -e^{-\alpha_j E_{j,t}[w_{j,t+1} - R_{f,t} w_{j,t}] + \frac{\alpha_j^2}{2} V_{j,t}[w_{j,t+1} - R_{f,t} w_{j,t}]} \\ &= -e^{-\alpha_j z_{j,t} E_{j,t}[R_{j,t+1}] + \frac{\alpha_j^2}{2} z_{j,t}^2 V_{j,t}[R_{j,t+1}]} \end{aligned} \tag{3.6}$$

then the optimal amount of investment in Bitcoin of period t is:

$$z_{j,t}^* = \frac{E_{j,t}\left[R_{t+1}\right]}{\alpha_j V_{j,t}\left[R_{t+1}\right]} \tag{3.7}$$

Equation (3.7) suggests that the amount of Bitcoin investment is independent to current level of wealth. When the Bitcoin investors expect a positive excess return in the next period, they will purchase Bitcoins; otherwise, they are going to sell Bitcoins. The two categories of Bitcoin investors, fundamentalists and speculators, use their individual rules to predict the price and make investment decisions. In the following subsections, price forecasting rules of these two Bitcoin investors are illustrated respectively.

3.4.2 Fundamentalist

Network effect, also known as network externality, describes the fact that the value of a product or service increases with the number of users. Examples of network effect include telephone system, softwares, and on-line social networks such as Facebook and Tencent. Network effect is present in Bitcoin network since it is a peer-to-peer payment network. Mefcalf's law quantifies the effect of network, or network externality, stating that the value of network is proportional to the square of its number of nodes. Therefore, Metcalf's law can be applied to Bitcoin network to capture the value of Bitcoin system. The fundamental value of Bitcoin system at period t, accordingly, is expressed as:

$$\tilde{\theta}_t = A_0 n_t^2 \tag{3.8}$$

Here, A_0 is a constant parameter, and n_t is the number of active Bitcoin users at period t which works as a proxy variable of number of nodes in Bitcoin system. Since whether Bitcoin is a mature functional technology or not remains controversial, $s \in \{0, 1\}$ is defined to represent the two status of Bitcoin system, in which $s = 1$ indicates that Bitcoin system is truly functional ("good"), and $s = 0$ implies Bitcoin system is defective ("bad"), followed by [32]. At the end of each period, Bitcoin system may collapse. In other words, the system will not survive and all the undergoing transactions will be lost. The probability of surviving is 1 for "good" status, and the probability of surviving for "bad" status is denoted as $1 - \chi$.

$$Pr(Collapse|s = 0) = \chi \in (0, 1) \tag{3.9}$$

$$Pr(Collapse|s = 1) = 0 \tag{3.10}$$

Equation (3.9) and (3.10) indicate that Bitcoin system can survive forever if it is truly a functional technology. However, the probability of collapse depends on the parameter χ if it has defects. In order to investigate the process of adopting Bitcoin system in the total population, $\bar{N}$ is defined as the number of total population, and Ω_t is denoted as the probability of public belief that Bitcoin system is "good" at period t, *i.e.*, $\Omega_t = Pr_t(s = 1)$.

Given Bitcoin system has survived until period t, then the probability that Bitcoin system will survive until period $t+1$ is:

$$\Omega_t \cdot 1 + (1-\Omega_t)(1-\chi) \tag{3.11}$$

The first item of equation (3.11), $\Omega_t \cdot 1$, is the probability that Bitcoin system survives in "good" status and the second item $(1-\Omega_t)(1-\chi)$ is the probability that Bitcoin system survives in "bad" status. Therefore, if Bitcoin system collapsed before period t, then $\Omega_i = 0, \forall i = t, ..., T$; if Bitcoin system survives till period t, then Ω_t satisfies the Bayes rule:

$$\Omega_t = \frac{\Omega_{t-1}}{\Omega_{t-1} + (1-\chi)(1-\Omega_{t-1})} = \frac{\Omega_0}{\Omega_0 + (1-\chi)^t(1-\Omega_0)} \tag{3.12}$$

Here Ω_0 is the initial public belief in Bitcoin system. Equation (3.12) suggests that the public belief in Bitcoin system is determined by the exogenous parameter χ. Conditional on $s = 1$, Ω_t increases monotonically and approaches to 1 with t.

The number of active users of Bitcoin is assumed to be the total number of populations who hold the belief that Bitcoin system is "good". Assuming the public belief is homogenous, the number of active Bitcoin users can be written as:

$$n_t = \Omega_t \bar{N} = \frac{\Omega_0 \bar{N}}{\Omega_0 + (1-\chi)^t(1-\Omega_0)} \tag{3.13}$$

In practice, beliefs may not only depend on the objective probabilities as dictated by the Bayes' rule, but also on an investor's knowledge of the number of existing participants in the Bitcoin trading market (See [80] and [81]). As is well known, a herd behavior may lead to sudden booms and crashes in the asset market, which may be offered as an alternative explanation for recent price movements of Bitcoin. This explanation would be germane if one assumes that Bitcoin are intrinsically useless. The present work takes an agnostic view of the intrinsic value of Bitcoin as a medium of exchange and also abstracts from herding behavior. The model instead posits network effects as a proxy for the fundamental value of Bitcoin as a medium of exchange.

In a more technically sophisticated model, both beliefs as well as the intrinsic value of Bitcoin may be allowed to depend on the number of users. However, as the fundamental value of Bitcoin already depends on the number of users, adding an additional network effect through beliefs will only add much complexity to the analyses without affecting the results qualitatively. This is left for future research.

By substituting equation (3.13) back into equation (3.8), the value of Bitcoin system can be represented as:

$$\tilde{\theta}_t = A_0 n_t^2 = A_0 \left(\frac{\Omega_0 \bar{N}}{\Omega_0 + (1-\chi)^t (1-\Omega_0)}\right)^2 \tag{3.14}$$

The fundamentalists have their own information-generating technologies. This technology allows each fundamentalist to obtain a noisy private signal $s_{\tilde{j},t}=\tilde{\theta}_t+\rho_{\tilde{j}}\varepsilon_{\tilde{j},t}$ at the beginning of each period t. The noisy information comes from a variety of sources such as the invention of alternative cryptic-currencies, the improvement in Bitcoin community and so on. In addition, $\tilde{\rho}_j$ is a positive parameter, $\varepsilon_{\tilde{j},t}$ is normalized to be uniformly distributed on [-1,1]. Conditional on $\tilde{\theta}_t$, the signals are independently and identically distributed for aggregate fundamentalists.

The fundamentalists predict the market price of Bitcoin in next period with referring to their private signals. They apply a threshold function of spot market price of Bitcoin $g(p_t)$ to help evaluate the private signals they receive. Specifically, the fundamentalists expect price of Bitcoin will rise in the next period if $s_{\tilde{j},t} \geq g(p_t)$. This condition can be re-written as $\varepsilon_{\tilde{j},t} \geq \frac{(g(p_t)-\tilde{\theta}_t)}{\rho_{\tilde{j}}}$.

Suppose all fundamentalists follow this rule, then the aggregate conditional mean of Bitcoin price in the next period is:

I. If $\tilde{\theta}_t < g(p_t) - \rho_{\tilde{J}}$, *i.e.*, all the fundamentalists receive the signals below the threshold, then the conditional mean is:

$$E_{\tilde{J},t}[p_{t+1}] = p_t + \tilde{\gamma_1}(\tilde{\theta}_t - g(p_t)) \tag{3.15}$$

II. If $\tilde{\theta}_t > g(p_t) + \rho_{\tilde{J}}$, *i.e.*, all the fundamentalists receive the signals above the threshold, then the conditional mean is:

$$E_{\tilde{J},t}[p_{t+1}] = p_t + \tilde{\gamma_2}(\tilde{\theta}_t - g(p_t)) \tag{3.16}$$

where $\tilde{J}$ represents the aggregation of fundamentalists. Both $\tilde{\gamma_1}$ and $\tilde{\gamma_2}$ are positive parameters with $\tilde{\gamma_2}$ greater than $\tilde{\gamma_1}$. This setting implies the fundamentalists realize speculators pursue short-term profits in the market. The extent to which fundamentalists are willing to increase the price forecast is higher than that to decrease the price forecast. In other words, the fundamentalists are conservative to lower their price predication when a passive signal is received.

III. If $g(p_t) - \rho_{\tilde{J}} \leq \tilde{\theta}_t \leq g(p_t) + \rho_{\tilde{J}}$, *i.e.*, some of the fundamentalists will forecast price increasing and the others will predict price decline, then the conditional mean of this situation becomes:

$$E_{\tilde{J},t}[p_{t+1}] = p_t + \frac{\tilde{\gamma_1} + \tilde{\gamma_2} + \left(\frac{g(p_t) - \tilde{\theta}_t}{\rho_{\tilde{J}}}\right)(\tilde{\gamma_1} - \tilde{\gamma_2})}{2}(\tilde{\theta}_t - g\left(p_t\right)) \tag{3.17}$$

Furthermore, for the fundamentalists, the conditional variance of price in the next period is defined in equation (3.18).

$$V_{\tilde{J},t}[p_{t+1}] = \tilde{\sigma_t}^2 \tag{3.18}$$

The aggregate fundamentalists are assumed to hold a homogeneous constant belief of conditional variance, and the second order effect is ignored here.

Finally, in order to solve the aggregate demand of fundamentalists explicitly, the specification of threshold function is set as a linear form:

$$g\left(p_t\right) = \tilde{\omega}_1 p_t \Phi_t + \tilde{\omega}_0 \tag{3.19}$$

Here, Φ_t is the aggregate transaction fee of Bitcoin network at period t which is denominated in Bitcoin. $\tilde{\omega}_0$ and $\tilde{\omega}_1$ are positive parameters. $p_t\Phi_t$ is the market cap of total transaction fee in Bitcoin system. For fundamentalists, they use this total transaction fee as a benchmark for the intrinsic value of Bitcoin system. If their private signals are higher than this benchmark, they will find more potential of Bitcoin system, such as the development of block chain technology application. If their private signals are lower than this benchmark, fundamentalists may receive information that is unfavourable to the valuation of Bitcoin system.

Substitute (3.15), (3.16), (3.17), (3.18) and (3.19) into (3.7), then the aggregate demand of fundamentalists $\tilde{J}$ at period t can be written as:

$$D^{\tilde{J},t} = \frac{\left(1 - \tilde{\gamma}_1 \tilde{\omega}_1 \Phi_t - R_{f,t}\right) p_t + \tilde{\gamma}_1 (\tilde{\theta}_t - \tilde{\omega}_0)}{\alpha_{\tilde{J}} \tilde{\sigma_t}^2} \quad for \ \frac{\tilde{\theta}_t - \tilde{\omega}_0 + \rho_{\tilde{J}}}{\tilde{\omega}_1 \Phi_t} < p_t; \tag{3.20}$$

$$D^{\tilde{J},t} = \frac{\left(1 - \tilde{\gamma}_2 \tilde{\omega}_1 \Phi_t - R_{f,t}\right) p_t + \tilde{\gamma}_2 (\tilde{\theta}_t - \tilde{\omega}_0)}{\alpha_{\tilde{J}} \tilde{\sigma_t}^2} \quad for \ \frac{\tilde{\theta}_t - \tilde{\omega}_0 - \rho_{\tilde{J}}}{\tilde{\omega}_1 \Phi_t} > p_t; \tag{3.21}$$

$$D^{\tilde{J},t} = \frac{\left(1 - R_{f,t}\right) p_t + \left(\frac{\tilde{\gamma}_1 + \tilde{\gamma}_2 + \left(\frac{\tilde{\omega}_0 + \tilde{\omega}_1 \Phi_t p_t - \tilde{\theta}_t}{\rho_{\tilde{J}}} \right) (\tilde{\gamma}_1 - \tilde{\gamma}_2)}{2} \right) \left(\tilde{\theta}_t - \tilde{\omega}_0 - \tilde{\omega}_1 \Phi_t p_t \right)}{\alpha_{\tilde{J}} \tilde{\sigma}^2}$$
$$for \ \frac{\tilde{\theta}_t - \tilde{\omega}_0 - \rho_{\tilde{J}}}{\tilde{\omega}_1 \Phi_t} \leq p_t \leq \frac{\tilde{\theta}_t - \tilde{\omega}_0 + \rho_{\tilde{J}}}{\tilde{\omega}_1 \Phi_t}. \quad (3.22)$$

It is notable that $\tilde{\theta}_t$ is defined in equation (3.14). The aggregate demand of fundamentalists described by above three equations reflect the fact that fundamentalists' behaviour are independent of market sentiment. Fundamentalists determine their investment strategies only based on relationship between received signals and the market prices.

3.4.3 Speculator

The essential feature of speculators is that they cannot and do not tend to collect information about the fundamental value of Bitcoin system. Since the market price of next period is uncertain, each speculator will choose one between two possible opinions on the market price (bullish & bearish), and the collection of these opinions turns into an overall opinion of speculators.

For a representative speculator $\hat{j}$ at time t, he or she will make profit if he or she predicts the overall expectation of Bitcoin market price in next period $t+1$ wisely. Recall that famous example- “Keynes links the stock market to beauty contest”. Guessing the taste of others instead of searching the objective beauty is the key in predicting the winner of contest.

Similarly, Bitcoin speculators predict the price of Bitcoin relying on their evaluation of the overall opinion of aggregate speculators. Therefore, the condition mean of market price in next period at time t is determined by Bitcoin speculator $\hat{j}$ as:

$$E_{\hat{j},t}[p_{t+1}] = p_t + \hat{\gamma} E_{\hat{j},t}[O_t] \tag{3.23}$$

where O_t is the overall opinion of aggregate Bitcoin speculators at period t. $\hat{\gamma}$ is a positive parameter which measures the impact of overall opinion on the price expectation. Equation (3.23) shows that speculars' forecast of price is an adjustment of spot price by expectation of overall opinion.

$\hat{J}_t$ is denoted as the total number of Bitcoin speculators at time t. $\hat{J}_t^+$ is the number of bullish Bitcoin speculators who believe market price will rise in the next period; $\hat{J}_t^-$ is the number of bearish speculators who believe the market price will decline in the next period. Therefore:

$$\hat{J}_t^+ + \hat{J}_t^- = \hat{J}_t \tag{3.24}$$

Following [73], the overall opinion O_t can be written as equation (3.25). $O_t > 0$ indicates that the overall opinion of Bitcoin speculators is bullish. In other word, more than half of speculators believe that price of the next period will increase; otherwise, the overall opinion of speculators is bearish.

$$O_t = \frac{\hat{J}_t^+ - \hat{J}_t^-}{\hat{J}_t} \in [-1, 1] \tag{3.25}$$

Suppose at the beginning of period t, each Bitcoin speculator may change his or her mind. $q_{t-1}^{\hat{j}+}$ is denoted as the probability that a bullish speculator $\hat{j}_{t-1}^+$ in previous period $t-1$ turn into bearish in period t, and $q_{t-1}^{\hat{j}-}$ is denoted as the probability that a bearish speculator $\hat{j}_{t-1}^-$ of previous period $t-1$ change to be bullish in current period t.

In addition, random variable $\nu_{\hat{j}+}$ represents the opinion of a previous bullish speculator, which takes 1 with a probability $q_{t-1}^{\hat{j}+}$ of switching to be bearish and 0 with probability $1-q_{t-1}^{\hat{j}+}$ of remaining to be bullish. Similarly, for the previous bearish speculator, the opinion is represented by random variable $\nu_{\hat{j}^-}$, which takes 1 with a probability $q_{t-1}^{\hat{j}-}$ of switching to be bullish and 0 of probability $1-q_{t-1}^{\hat{j}-}$ of remaining to be bearish.

Given probabilities $q_{t-1}^{\hat{j}+}$ and $q_{t-1}^{\hat{j}-}$, variables $\nu_{\hat{j}^+}$ and $\nu_{\hat{j}^-}$ are *i.i.d.* Through collecting the opinions of all speculators, the number of Bitcoin bullish speculators in period t is:

$$\hat{J}_t^+ = \sum_{\hat{j}_{t-1}^+=1}^{\hat{J}_{t-1}^+} \left(1 - \nu_{\hat{j}^+}\left(q_{t-1}^{\hat{j}+}\right)\right) + \sum_{\hat{j}_{t-1}^-=1}^{\hat{J}_{t-1}^-} \nu_{\hat{j}^-}\left(q_{t-1}^{\hat{j}-}\right) \tag{3.26}$$

On the right side of (3.26), the first item is the number of speculators who are bullish at period $t-1$ and remain bullish till period t. The second item represents the speculators who are bearish at $t-1$ but switch to be bullish in period t. Both of them consist speculators who are bullish in period t.

Similarly, the number of Bitcoin bearish speculator at period t can be formulated as:

$$\hat{J}_t^- = \sum_{\hat{j}_{t-1}^+=1}^{\hat{J}_{t-1}^+} \left(\nu_{\hat{j}^+}\left(q_{t-1}^{\hat{j}+}\right)\right) + \sum_{\hat{j}_{t-1}^-=1}^{\hat{J}_{t-1}^-} (1 - \nu_{\hat{j}^-}\left(q_{t-1}^{\hat{j}-}\right)) \tag{3.27}$$

On the right side of (3.27), the first item is the number of speculators who were bullish at $t-1$ but switch to be bearish in period t. The second item represents the number of speculators who are bearish at $t-1$ and still bearish in period t. Both of them consist bearish speculators in period t. After the numbers of different types of speculators at each t are obtained, the overall opinion of speculator is calculated by substituting (3.26) and (3.27) into (3.25):

$$O_t = \frac{\sum_{\hat{j}_{t-1}^+=1}^{\hat{J}_{t-1}^+}\left[1 - 2\nu_{\hat{j}^+}\left(q_{t-1}^{\hat{j}+}\right)\right] + \sum_{\hat{j}_{t-1}^-=1}^{\hat{J}_{t-1}^-}\left[2\nu_{\hat{j}^-}\left(q_{t-1}^{\hat{j}-}\right) - 1\right]}{\hat{J}_t} \tag{3.28}$$

where $\hat{J}$ is assumed to be $>> 1$. In other words, an individual speculator $\hat{j}$'s opinion will not have a impact on the overall opinion.

Since $\nu_{\hat{j}^+}$ and $\nu_{\hat{j}^-}$ are *i.i.d*, the mean of aggregate Bitcoin speculators'opinion is:

$$E_{\hat{j},t}\left[O_t\right] = O_{t-1} + q_{t-1}^{\hat{j}-}\left(1 - O_{t-1}\right) - q_{t-1}^{\hat{j}+}\left(1 + O_{t-1}\right) \tag{3.29}$$

In addition, the probabilities of switching $q_{t-1}^{\hat{j}\pm}$ are set as the functions of overall opinion and price deviation:

$$q_{t-1}^{\hat{j}-}\left(O_{t-1}, G_{t-1}\right) = \psi_0 + \psi_1 O_{t-1} + \psi_2 G_{\hat{j},t-1} - \psi_3 G_{\hat{j},t-1}^2 \tag{3.30}$$

$$q_{t-1}^{\hat{j}+}\left(O_{t-1}, G_{t-1}\right) = \psi_0 - \psi_1 O_{t-1} - \psi_2 G_{\hat{j},t-1} + \psi_3 G_{\hat{j},t-1}^2 \tag{3.31}$$

where $G_{\hat{j},t-1} = p_{t-1} - \mu_{\hat{j},t-1}$ is the deviation from the observed price to the sample of previous prices. ψ_0, ψ_1, ψ_2, ψ_3 are positive parameters. By setting ψ_0, ψ_1, ψ_2, ψ_3 to be small enough, $q_{t-1}^{\hat{j}\pm}$ is restricted into interval $[0, 1]$.

In equations (3.30) and (3.31), $\frac{\partial q_{t-1}^{\hat{j}-}}{\partial O_{t-1}} = \psi_1 > 0$ and $\frac{\partial q_{t-1}^{\hat{j}+}}{\partial O_{t-1}} = -\psi_1 < 0$. The bearish (bullish) speculator tends to be bullish (bearish) if the aggregate opinion is bullish (bearish), which captures the characteristics of social imitation among speculators.

Additionally, in equation (3.30), $\frac{\partial q_{t-1}^{\hat{j}-}}{\partial G_{\hat{j},t-1}} = \psi_2 - 2\psi_3 G_{\hat{j},t-1} > 0$ when $G_{\hat{j},t-1} < \frac{\psi_2}{2\psi_3}$, *i.e.*, $p_{t-1} < \mu_{\hat{j},t-1} + \frac{\psi_2}{2\psi_3}$, and $\frac{\partial q_{t-1}^{\hat{j}-}}{\partial G_{\hat{j},t-1}} = \psi_2 - 2\psi_3 G_{\hat{j},t-1} < 0$ when $G_{\hat{j},t-1} > \frac{\psi_2}{2\psi_3}$, *i.e.*, $p_{t-1} > \mu_{\hat{j},t-1} + \frac{\psi_2}{2\psi_3}$. The probability that previous bearish speculator switches to be bullish increases with the price deviation increasing when the observed price deviation is below the threshold $\frac{\psi_2}{2\psi_3}$. Meanwhile, when the price deviation is above the threshold, the probability that previous bearish speculator switches to be bullish decreases with the price deviation increasing.

On the other side, in equation (3.31), $\frac{\partial q_{t-1}^{\hat{j}+}}{\partial G_{\hat{j},t-1}} = -\psi_2 + 2\psi_3 G_{\hat{j},t-1} > 0$ when $G_{\hat{j},t-1} > \frac{\psi_2}{2\psi_3}$, *i.e.*, $p_{t-1} > \mu_{\hat{j},t-1} + \frac{\psi_2}{2\psi_3}$, and $\frac{\partial q_{t-1}^{\hat{j}+}}{\partial G_{\hat{j},t-1}} = -\psi_2 + 2\psi_3 G_{\hat{j},t-1} < 0$ when $G_{\hat{j},t-1} < \frac{\psi_2}{2\psi_3}$, *i.e.*, $p_{t-1} < \mu_{\hat{j},t-1} + \frac{\psi_2}{2\psi_3}$. Therefore, when the observed price deviation is above the threshold $\frac{\psi_2}{2\psi_3}$, the probability that previous bullish speculator switches to bearish increases with increasing of price deviation. When the observed price deviation is below the threshold $\frac{\psi_2}{2\psi_3}$, the probability that previous bullish speculator switches to bearish increases with the decline of price deviation.

Moreover, speculator $\hat{j}$ is assumed to apply geometric distribution to determine his sample mean, which reflects the characteristics of trend following.

$$\mu_{\hat{j},t-1} = \frac{\delta_{\hat{j}}}{1 - \delta_{\hat{j}}} \sum_{i=1}^{L} \delta_{\hat{j}}^{\,i-1} p_{t-i} \tag{3.32}$$

where $\delta_{\hat{j}}$ is the memory rate of geometric probability distribution, which works as weight for previous prices. L is the indicator of lagging periods.

Larger L implies that longer periods of previous Bitcoin price to generate the sample mean. Substitute (3.30) and (3.31) into (3.29), then:

$$E_{\hat{j},t}\left[O_t\right] = (1 + 2\psi_1 - 2\psi_0)O_{t-1} + 2\psi_2 G_{\hat{j},t-1} - 2\psi_3 G^2_{\hat{j},t-1} \tag{3.33}$$

By combing (3.33), (3.23) and (3.32), equation of speculator $\hat{j}$'s the condition mean is:

$$E_{\hat{j},t}[p_{t+1}] = p_t + (1 + 2\psi_1 - 2\psi_0)\,\hat{\gamma}O_{t-1} + \\ 2\psi_2\hat{\gamma}\left(p_{t-1} - \frac{\delta_{\hat{j}}}{1-\delta_{\hat{j}}}\sum_{i=1}^{L}\delta_{\hat{j}}^{i-1}p_{t-i}\right) - 2\hat{\gamma}\psi_3(p_{t-1} - \frac{\delta_{\hat{j}}}{1-\delta_{\hat{j}}}\sum_{i=1}^{L}\delta_{\hat{j}}^{i-1}p_{t-i})^2 \tag{3.34}$$

Furthermore, speculator $\hat{j}$ determines the condition variance of Bitcoin price in next period as follows:

$$V_{\hat{j},t}[p_{t+1}] = \eta + \hat{\rho}\sigma_{\hat{j},t}^2 \tag{3.35}$$

where $\sigma_{\hat{j},t}^2 = \frac{\delta_{\hat{j}}}{1-\delta_{\hat{j}}}\sum_{i=1}^{L}\delta_{\hat{j}}^{i-1}\left(p_{t-i} - \mu_{\hat{j},t-i}\right)^2$, which is the sample variance determined by speculator $\hat{j}$. η and $\hat{\rho}$ are non-negative constant parameters. When L tends to be infinite, the sample mean and variance become:

$$\mu_{\hat{j},t} = \delta_{\hat{j}}\mu_{\hat{j},t-1} + (1-\delta_{\hat{j}})p_t \tag{3.36}$$

$$\sigma_{\hat{j},t}^2 = \delta_{\hat{j}}\sigma_{\hat{j},t-1}^2 + \delta_{\hat{j}}(1-\delta_{\hat{j}})\left(p_t - \mu_{\hat{j},t}\right)^2 \tag{3.37}$$

Assume that the group of speculators is homogeneous, then the aggregate demand of speculators $\hat{J}$ at period t is written as:

$$D^{\hat{J},t} = \frac{(1-R_{f,t})\,p_t + (1+2\psi_1-2\psi_0)\hat{\gamma}O_{t-1} + 2\hat{\gamma}\psi_2 G_{\hat{J},t-1} - 2\hat{\gamma}\psi_3 G_{\hat{J},t-1}^2}{\alpha_{\hat{J}}\eta\left(1+\hat{\rho}\sigma_{\hat{J},t}^2\right)} \tag{3.38}$$

3.4.4 Dynamic of Market Price

Consider a typical period in which one Bitcoin block is generated, J_t is denoted as the difference between the number of fundamentalists and speculators in period t, *i.e.*, $J_t = \tilde{J}_t - \hat{J}_t$ and J_t is standardized into interval $[-1,1]$. $J_t = 1(-1)$ indicates that all investors in the market are fundamentalists (speculators). The weighted aggregate excess demand of Bitcoin can be written as $\frac{1+J_t}{2} D^{\tilde{J},t} + \frac{1-J_t}{2} D^{\hat{J},t}$.

Exogenous Q_t Bitcoins are mined out in this period. In order to absorb excess demand (supply), market maker offset the positions in Bitcoin. After all the transactions are executed, price of Bitcoin in next period $t+1$ satisfies:

$$p_{t+1} - p_t = \kappa(\frac{1+J_t}{2} D^{\tilde{J},t} + \frac{1-J_t}{2} D^{\hat{J},t} - Q_t) \tag{3.39}$$

Here κ is positive parameter which measures the sensitivity of market price to change in demand (supply). If the aggregate effect of all speculators' behaviour is to sell, then the price of next period p_{t+1} will decrease, *i.e.*, $p_{t+1} < p_t$; If the aggregate effect is to purchase, then the price in next period p_{t+1} will increase, *i.e.*, $p_{t+1} \geq p_t$. The direction of price change is always in inhibiting excess demand (supply).

Moreover, speculation on Bitcoin is also impacted by external shocks, then ε_t, which follows normal distribution, *i.e.* $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$, is added to the path of price, in order to make the equation more realistic.

$$p_{t+1} = p_t + \kappa(\frac{1+J_t}{2} D^{\tilde{J},t} + \frac{1-J_t}{2} D^{\hat{J},t} - Q_t) + \varepsilon_t \tag{3.40}$$

Substitute (3.20) (3.21), (3.22) and (3.38) into (3.40), then the market price of Bitcoin is determined by the equation system (3.41).

$$
\begin{aligned}
p_{t+1} &= p_t + \kappa(\frac{1+\hat{J}_t}{2}(\frac{(1-\tilde{\gamma}_1\tilde{\omega}_1\Phi_t - R_{f,t})\,p_t + \tilde{\gamma}_1(\tilde{\theta}_t - \tilde{\omega}_0)}{\alpha_{\tilde{J}}\tilde{\sigma}^2})+ \\
&\frac{1-\hat{J}_t}{2}(\frac{(1-R_{f,t})\,p_t + (1+2\psi_1-2\psi_0)\hat{\gamma}O_{t-1} + 2\hat{\gamma}\psi_2 G_{\hat{J},t-1} - 2\hat{\gamma}\psi_3 G^2_{\hat{J},t-1}}{\alpha_{\hat{J}}\eta\left(1+\hat{\rho}\sigma^2_{\hat{J},t}\right)}) - Q_t) + \varepsilon_t \\
&if\ \ \frac{\tilde{\theta}_t - \tilde{\omega}_0 + \rho_{\tilde{J}}}{\tilde{\omega}_1\Phi_t} < p_t; \\
&= p_t + \kappa(\frac{1+\hat{J}_t}{2}(\frac{(1-\tilde{\gamma}_2\tilde{\omega}_1\Phi_t - R_{f,t})\,p_t + \tilde{\gamma}_2(\tilde{\theta}_t - \tilde{\omega}_0)}{\alpha_{\tilde{J}}\tilde{\sigma}^2})+ \\
&\frac{1-\hat{J}_t}{2}(\frac{(1-R_{f,t})\,p_t + (1+2\psi_1-2\psi_0)\hat{\gamma}O_{t-1} + 2\hat{\gamma}\psi_2 G_{\hat{J},t-1} - 2\hat{\gamma}\psi_3 G^2_{\hat{J},t-1}}{\alpha_{\hat{J}}\eta\left(1+\hat{\rho}\sigma^2_{\hat{J},t}\right)}) - Q_t) + \varepsilon_t \\
&if\ \ \frac{\tilde{\theta}_t - \tilde{\omega}_0 - \rho_{\tilde{J}}}{\tilde{\omega}_1\Phi_t} > p_t; \\
&= p_t + \kappa(\frac{1+\hat{J}_t}{2}(\frac{(1-R_{f,t})\,p_t + \left(\frac{\tilde{\gamma}_1+\tilde{\gamma}_2+\left(\frac{\tilde{\omega}_0+\tilde{\omega}_1\Phi_t p_t-\tilde{\theta}_t}{\rho_{\tilde{J}}}\right)(\tilde{\gamma}_1-\tilde{\gamma}_2)}{2}\right)\left(\tilde{\theta}_t - \tilde{\omega}_0 - \tilde{\omega}_1\Phi_t p_t\right)}{\alpha_{\tilde{J}}\tilde{\sigma}^2})+ \\
&\frac{1-\hat{J}_t}{2}\cdot(\frac{(1-R_{f,t})\,p_t + (1+2\psi_1-2\psi_0)\hat{\gamma}O_{t-1} + 2\hat{\gamma}\psi_2 G_{\hat{J},t-1} - 2\hat{\gamma}\psi_3 G^2_{\hat{J},t-1}}{\alpha_{\hat{J}}\eta\left(1+\hat{\rho}\sigma^2_{\hat{J},t}\right)}) - Q_t) + \varepsilon_t \\
&if\ \ \frac{\tilde{\theta}_t - \tilde{\omega}_0 - \rho_{\tilde{J}}}{\tilde{\omega}_1\Phi_t} \le p_t \le \frac{\tilde{\theta}_t - \tilde{\omega}_0 + \rho_{\tilde{J}}}{\tilde{\omega}_1\Phi_t}.
\end{aligned}
\tag{3.41}
$$

Till this moment, we have completed the model of Bitcoin market price. Fundamentalists apply their collected information and individual valuation methods to evaluate intrinsic value of Bitcoin. Speculators forecast the price through applying a non-linear function to measure the impact of public opinion and trend of historical prices. Their interaction determines the dynamic path of market price. In Section 5, the model will be calibrated.

3.5 Numerical Simulation & Empirical Analysis

The purpose of Monte Carlo simulation is to explore the evolution of fundamental value of Bitcoin system and the potential sources of high volatility in Bitcoin market price. The statistical properties of simulated and historical market price series will be compared in the following empirical analysis.

3.5.1 Numerical Simulation

Following the parameter setting in [32], Ω_0 is set to be 0.005. $\bar{N}$ is set as 10,000. The logarithmic monthly data of Bitcoin system market cap from July 16^{st}, 2010 to December 31^{st}, 2012 are selected to fit equation (3.14), $\tilde{\theta}_t = A_0(\frac{\Omega_0 \bar{N}}{\Omega_0+(1-\chi)^t(1-\Omega_0)})^2$, to determine the parameter A_0 and χ. The logarithmic form of equation (3.14) can be written as: $log\tilde{\theta}_t = logA_0 + 2log(\Omega_0 \bar{N}) - 2log(\Omega_0 + (1-\chi)^t(1-\Omega_0))$. The reason of selecting this sample is to exclude the data which might involve speculative behaviour after 2012. The estimated A_0 and χ are 0.783 and 0.100, respectively.

Besides, given the monthly data of market cap, the historical market price p_t and the transaction fee Φ_t denominated in Bitcoin in the same period, the coefficients $\tilde{\omega}_0$, $\tilde{\omega}_1$ in threshold equation (3.19), $g(p_t) = \tilde{\omega}_1 p_t \Phi_t + \tilde{\omega}_0$, are estimated. The estimation results are summarized in Table 3.1. The adjusted R-square is 0.577 in Table 3.1 and p-values of both coefficients are strongly significant, hence the estimation result is reasonable.

The logarithmic monthly market cap of Bitcoin system, the aggregate revenue of transaction fee and the intrinsic value of Bitcoin system calculated by estimated A_0 from July 1^{st}, 2010 to December 31^{st}, 2012 are displayed in Figure 3.3. The moving patterns of the market cap and the aggregate revenue of transaction fee were similar, which indicates that Bitcoin serving as an exchange medium drives the market value to increase.

For the simulation, the parameters for the fundamentalists $\tilde{\gamma}_1$ and $\tilde{\gamma}_2$ are set as 0.02 and 0.05 respectively, which satisfies $\tilde{\gamma}_1 < \tilde{\gamma}_2$. Besides, the parameter $\tilde{\rho}_J$ is set as 0.1, which provides the proper disturbance to the received signal. Following [60] and [79], for the speculators, $\hat{\gamma}$ in equation (3.23) is set as 0.3. Additionally, both $\hat{\rho}$ and η are set as 0.5 in order to construct the sample variance

Table 3.1 Regression of the Threshold Function

Regression Statistics					
Multiple R	7.693e-01				
R Square	5.918e-01				
Adjusted R Square	5.772e-01				
Standard Error	7.714e-01				
Observation	30				
ANOVA					
	df	SS	MS	F	Significance F
Regression	1	24.158	24.158	40.596	0.000
Residual	29	14.662	5.951e-01		
Total	30	40.820			
	Coefficients	Standard Error	P-Value	Lower 95%	Upper 95%
$\tilde{\omega}_0$	4.037	1.645e-01	0.000	3.700	4.374
$\tilde{\omega}_1$	5.718e-01	8.975e-02	0.000	3.880e-01	7.557e-01

of the speculators. Moreover, for equations (3.30) and (3.31), the parameters ψ_0, ψ_1, ψ_2, ψ_3 are set as 0.3, 0.03, 0.003 and $6.250e^{-6}$ respectively, which satisfies that the probability of switching is restricted to the interval [0,1]. Furthermore, $\alpha_{\hat{j}}$ equals 0.1 and the memory rate of geometric probability distribution $\delta_{\hat{j}}$, is set as 0.85. Finally, the parameter $\tilde{\omega}_0$, $\tilde{\omega}_1$ and A_0 estimated are used in the simulation.

The rest of the market structure parameters are set as follows. The daily risk-free interest rate $R_{f,t}$ is set as 1.0001, *i.e.*, 1+0.01% which indicates that the annual interest rate equals to 3.717% approximately. Market fraction J is restricted within the interval [-1,-0.5], which indicates that at least three quarters of investors in each period are the speculators. κ, which reflects the depth of market, is set as 0.002. In addition, the initial value of $\tilde{\sigma}_t^2$ is the same as $\hat{\sigma}_t^2$ for the variance of the expected return rate is set as 1. Moreover, the initial value of the opinion index O_t is set as 0, which implies the aggregate speculators are unbiased at the starting point. Furthermore, the external shock ϵ_t follows a standard normal distribution $N(0,1)$. Finally, the path of the extra Bitcoin exogenous supply is normalized as $0.005 - 0.0001t$ and converges to zero after 50 time periods.

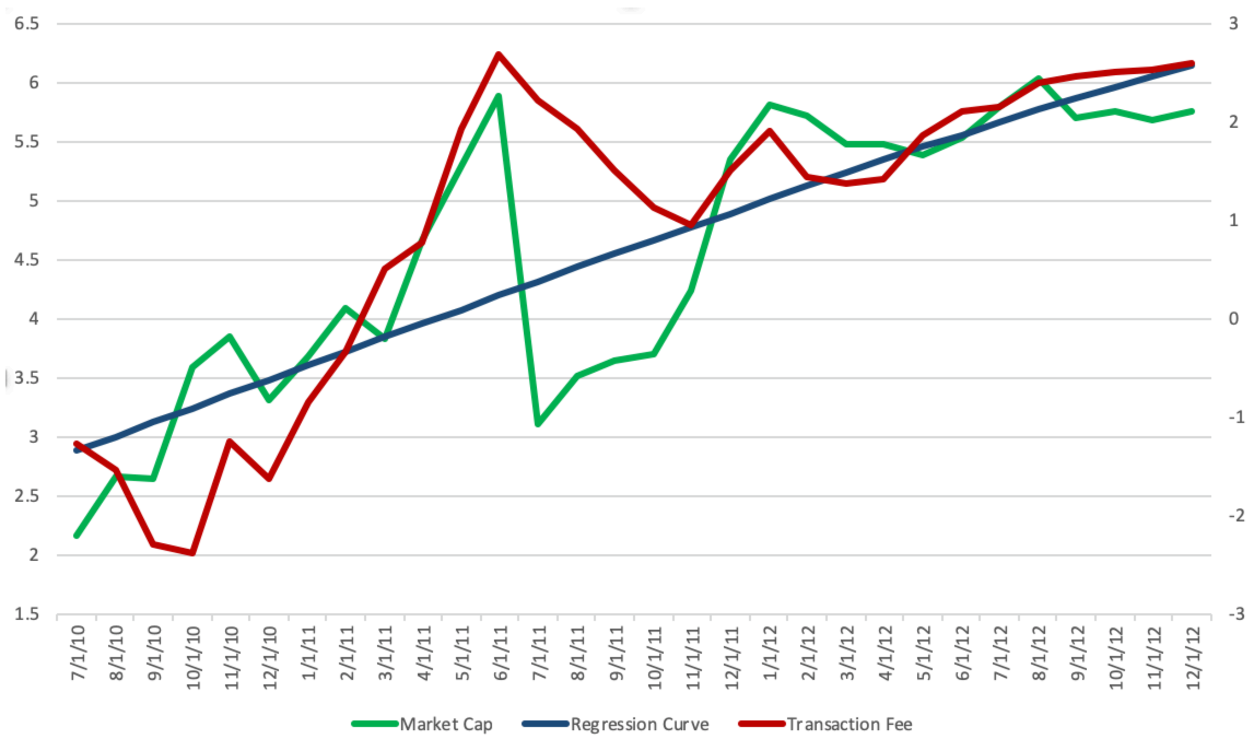


Figure 3.3 Regression Fitness of Logarithm of Bitcoin Market Cap

The length of the generated data is 3100 time periods and the number of the simulation times is 100. Figure 3.4 displays these 100 simulated daily market prices of Bitcoin. After a steady rise in the very beginning, the curves are followed by a sharp plunge immediately. Generally, a re-bounce would appear after the decline but they would not come back to the previous peak value. The simulated market price of Bitcoin in Figure 3.4 and the settings of parameters work as a benchmark for the additional analysis in pricing mechanism.

In order to study the individual impacts from the fundamentalists and speculators, the situations where the market is full of the fundamentalists and that is full of the speculators will be investigated separately. First, Figure 3.5 displays the evolution of the fundamental values of Bitcoin system with different probabilities of collapse χ. With other parameters unchanged, χ is set as 0.1, 0.01, 0.004 and 0.001 respectively. In Figure 3.5, given Bitcoin system survives in the whole

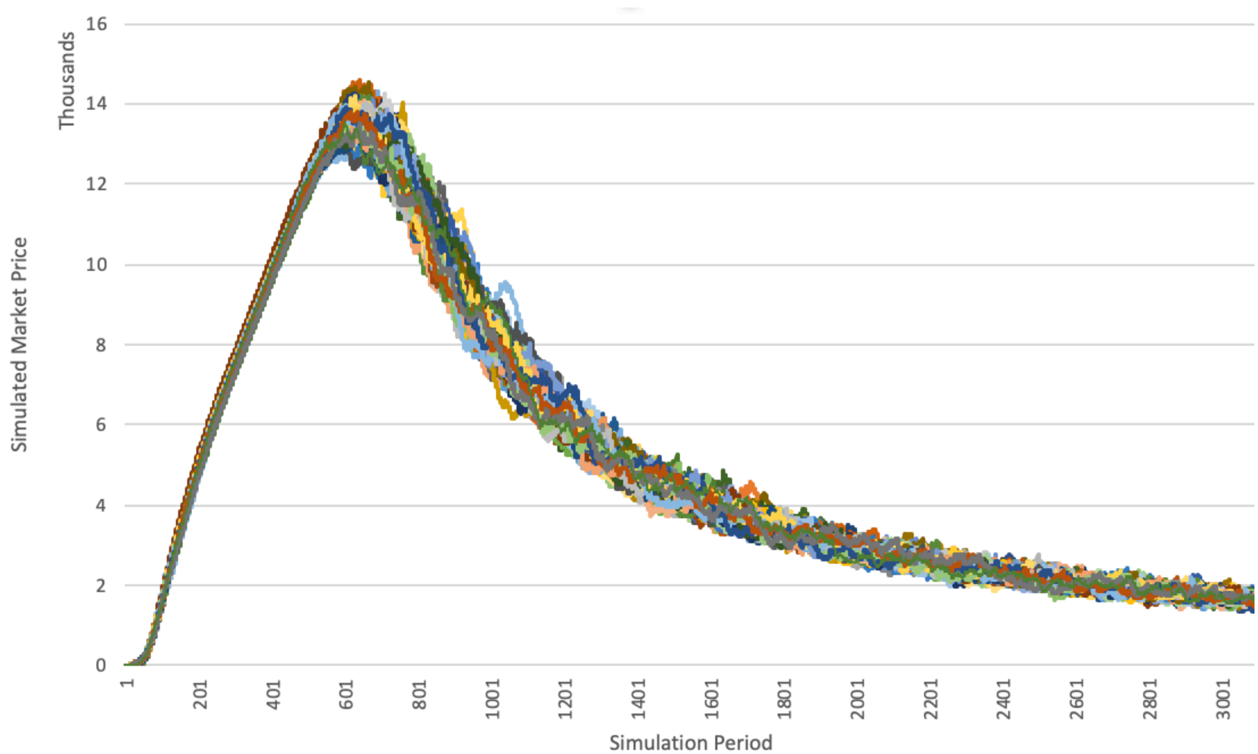


Figure 3.4 Simulation of Bitcoin Market Price Series

sample period, a higher χ will cause Bitcoin system to converge to its fundamental value faster. In other words, given a higher probability of collapse, the public belief in Bitcoin system will be reinforced when Bitcoin system maintains the status as survival.

Besides, in Figure 3.6, the evolution of the simulated market prices in the case of pure fundamentalists are displayed. There are four families of curves with different χ included in Figure 3.6. The family of the highest market prices of Bitcoin appear when $\chi = 0.1$. The peaks of other curves, which are generated with smaller χ, are lower than that of the curves with $\chi = 0.1$, and these peaks appear later than that of the curves in case $\chi = 0.1$. Therefore, if the market is consisted of only fundamentalists and Bitcoin system always survives during the whole sample period, a larger probability of collapse will cause Bitcoin system to converge to its fundamental value faster, and incite the fundamentalist to accept a peak of higher market price.

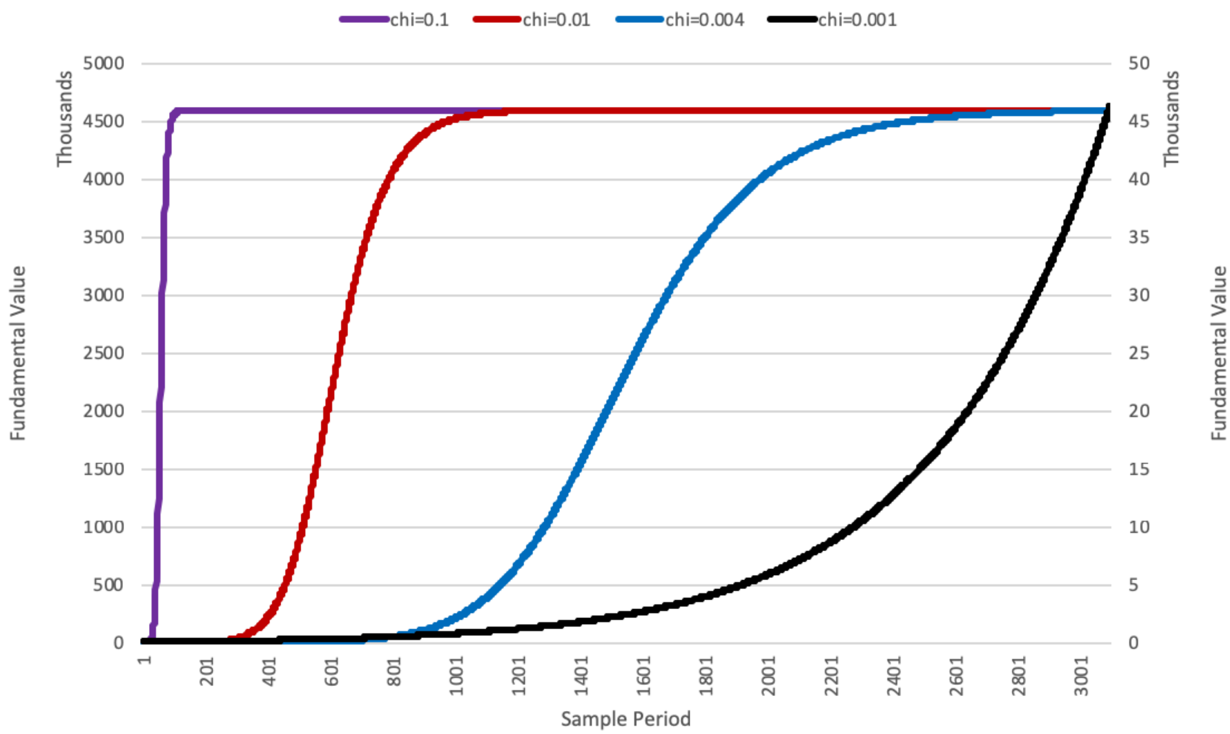


Figure 3.5 Fundamental Values of Bitcoin System under Different χ

How the different probabilities of collapse χ affect the speculators' behaviour is showed in Figure 3.7. By comparing Figures 3.6 and 3.7, the highest market prices and maximum magnitude of price enlarge are obtained with the highest χ. In Figure 3.7, for the cases with $\chi = 0.01$, $\chi = 0.004$ and $\chi = 0.001$, all three families of market price series have some extent of overlap, which is the evidence of non-linear trend following characteristics of speculators.

Consider the other situation in which all the market agents are speculators. The evolution of the aggregate opinion index of the speculators in this case is exhibited in Figure 3.8. In plot (a) of Figure 3.8, the path of opinion index always stays as zero which indicates the attitude of aggregate speculators is always neutral without the external shock (information) ($\varepsilon_t = 0$). The variances of ε_t in equation (3.40) are equal to $1, 5, 10$ in plot (b), (c) and (d) respectively. With increase in σ_ϵ^2, the possible range of the opinion index becomes larger and more negative. It implies that the spec-

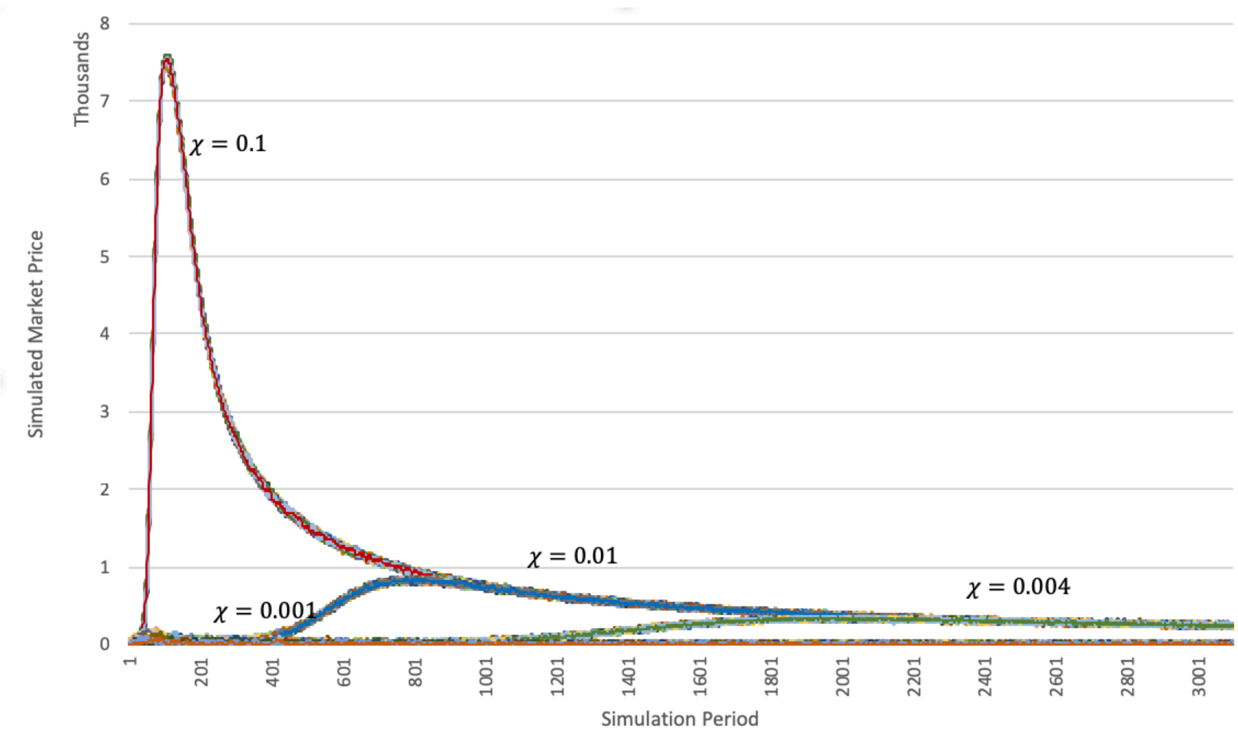


Figure 3.6 Market Prices of Bitcoin under Different χ: Case of Pure Fundamentalists

ulators tend to be bearish facing volatile random shocks, given no information of the fundamental value is available.

The corresponding market prices in the case of pure speculators with the different external shocks are displayed in Figure 3.9. In plot (a) of Figure 3.9, all the 100 market prices are restricted to zero, given the initial value of price set as zero. In plots (b), (c) and (d), similar to the paths of the aggregated opinion, the range of the market price series also enlarges with increasing σ_ϵ^2 which implies that the stochastic external shock incites the speculating behaviour. In these simulations, the minimum market price is set as zero indicating no transaction will be made among the speculators.

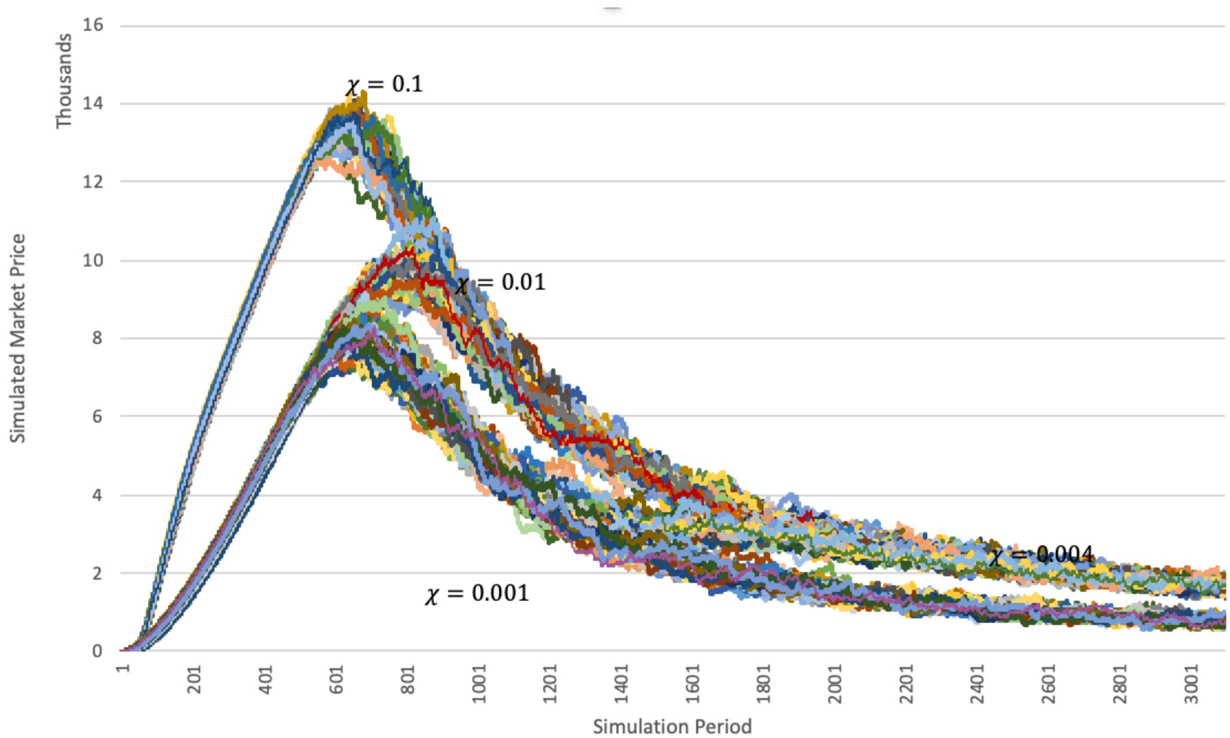
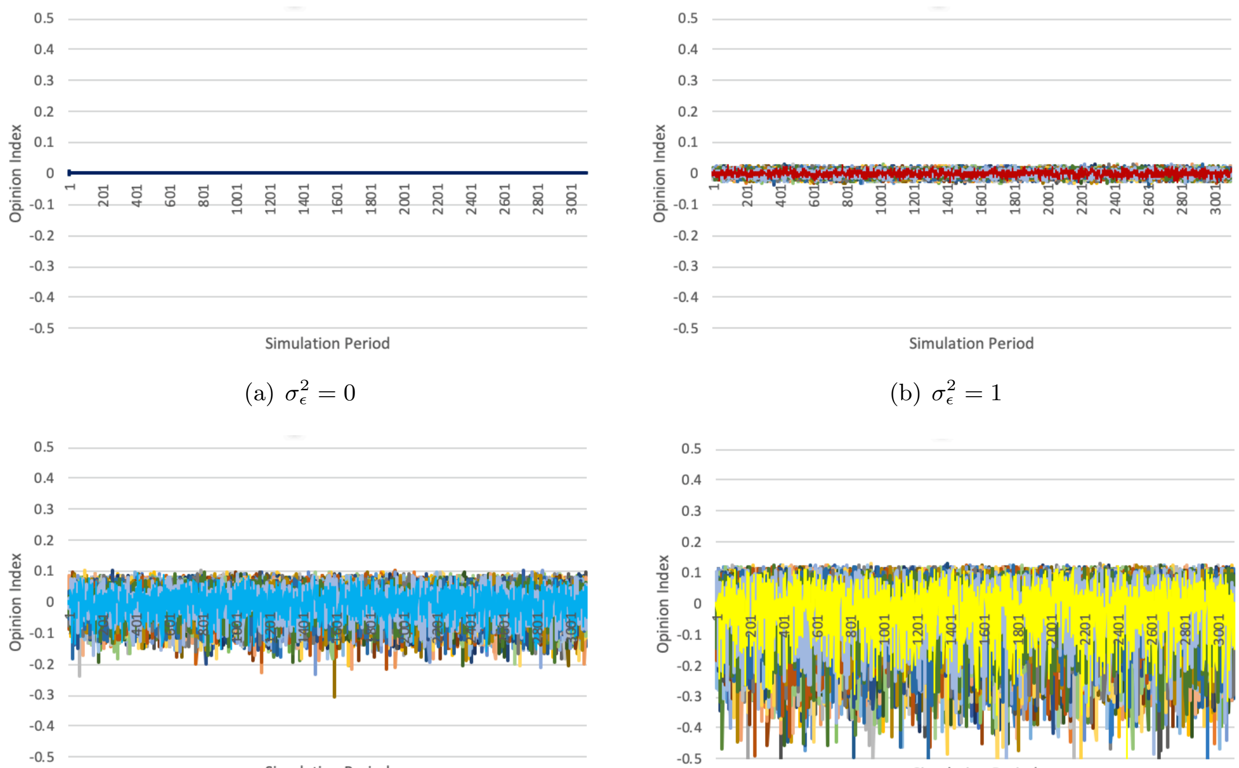


Figure 3.7 Simulated Market Price under Different χ

Overall, the excess demand of the fundamentalists provides the baseline of Bitcoin market price which is determined by the different probabilities of collapse. Without the fundamentalists, the trading of the pure speculators leads to the random process of market prices. By comparing with the series in Figure 3.4 and the series of $\chi = 0.1$ in Figure 3.6, under the interactions of fundamentalists and speculators, the fluctuation of market price becomes larger.

How the behaviour of speculators results in the volatility of market prices is investigated by adjusting the market fraction J_t. The interval of value J_t, [-1,1], is divided into eight sub-intervals evenly. The larger proportion of speculators appears when J_t is close to -1 and the larger proportion of fundamentalists exists when J_t is close to 1.



(a) $\sigma_\epsilon^2 = 0$

(b) $\sigma_\epsilon^2 = 1$

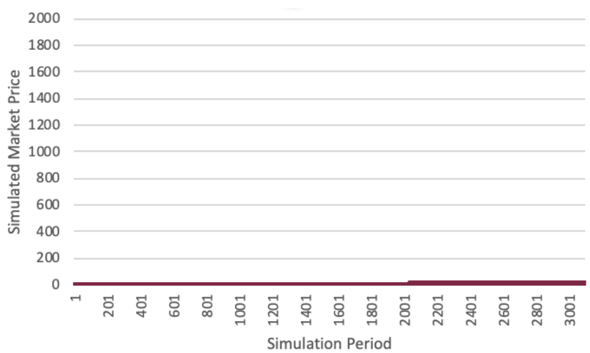
(c) $\sigma_\epsilon^2 = 5$

(d) $\sigma_\epsilon^2 = 10$

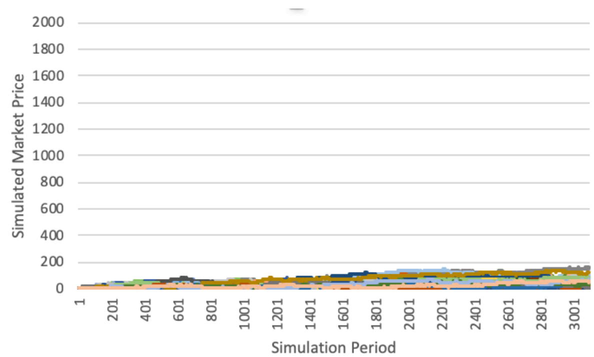
Figure 3.8 Opinion Index of Aggregate Speculators under Different External Shock

In Figures 3.10 and 3.11, different families of market price series with various market fractions J_t are displayed. For each sub-interval, the value of J_t is drawn randomly from the uniform distribution. Given all other parameters unchanged, the highest market price is obtained in the blue curve of Figure 3.10 which represents the case with largest proportion of speculators ([-1,-0.75]). It can be concluded that the behaviour of speculators brings the volatility of price series. By comparing Figure 3.10 and Figure 3.11, with fewer speculators in the market, the market price is closer to the case of pure fundamentalists.

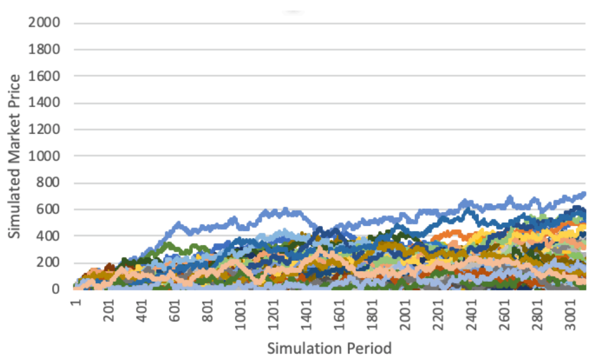
Auto-correlation (AC) of the simulated prices is explored as well in order to analyze the path dependence of speculators' strategies. The logarithmic return of the market price r_t is defined as



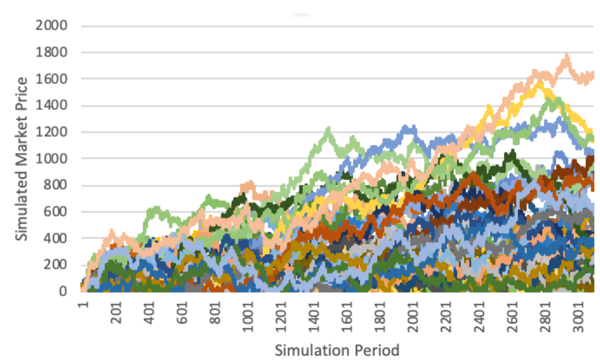
(a) $\sigma_\epsilon^2 = 0$



(b) $\sigma_\epsilon^2 = 1$



(c) $\sigma_\epsilon^2 = 5$



(d) $\sigma_\epsilon^2 = 10$

Figure 3.9 Market Price in Case of Pure Speculators under Different External Shock

$r_t = ln\ p_t - ln\ p_{t-1}$. For the all 100 independently simulated price series, the AC of return rates r, the absolute return rates $|r|$ and the squared return rates r^2 are estimated separately, and the average of the coefficients is calculated at each time point. The estimated ACs and the corresponding confidence intervals are displayed in Figures 3.12 - 3.14 separately. The long-term dependence in return rates has been broadly studied since [82], who investigated the AC of daily S&P 500 index from 1928 to 1991. This study finds that the absolute and squared return rate series tend to have very slow decay AC. In addition, the AC of absolute return rates at each lag is greater than that of the squared return rates till at least 100 lags.

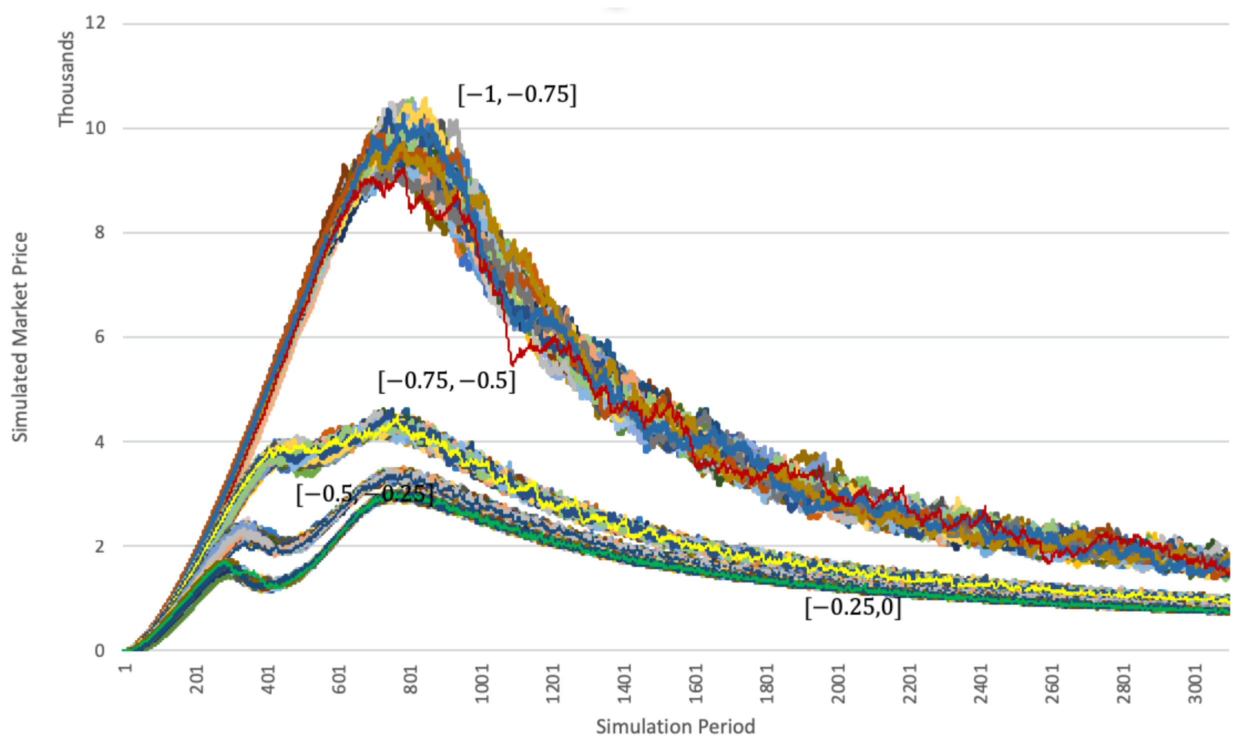


Figure 3.10 Market Prices with Different Fractions: Case of $J < 0$

In Figure 3.12, the AC coefficients are significant till lag 8 for the simulated return rate series. For the absolute and squared simulated return rate series, both the sample correlation coefficients are outside the 95% confidence interval and stay positive for the long lagging order. Besides, all the sample AC of the absolute return rate series are greater than that of the squared return rate series at each lag for up to 80 lags. This long-term dependence is resulted from the effect of herding and social imitation of the speculators on the aggregate market price series.

In the following sub-section, the statistical methodologies are adopted to estimate the historical market price and the simulated price series, in order to compare the statistical properties between the actual data and the data generated from the theoretical model.

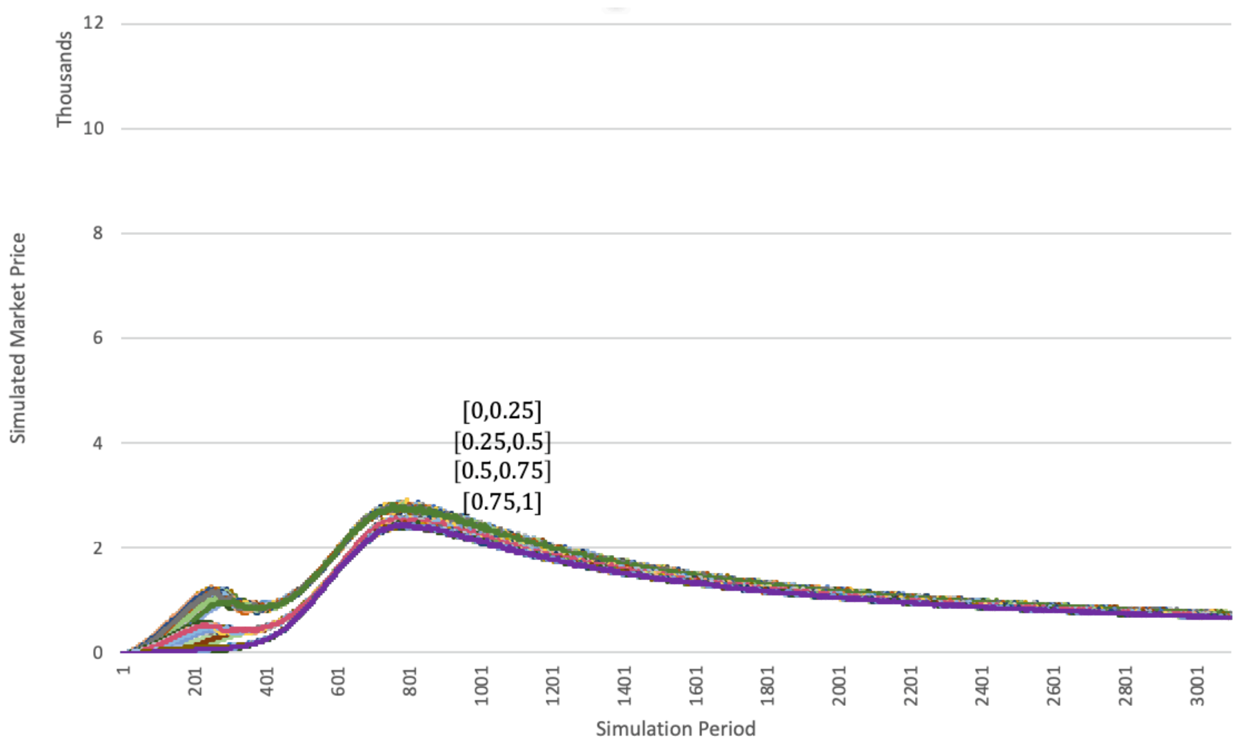


Figure 3.11 Market Prices with Different Fractions Case of $J > 0$

3.5.2 Empirical Analysis

In this section, a statistical analysis of the historical Bitcoin market price and the simulated price is performed. The purpose of the empirical analysis is to investigate whether the market price of Bitcoin reflects the same characteristics of speculating as in the traditional financial market and whether the price series simulated from the theoretical model have the similar statistical properties to the historical price series. For this empirical analysis, a total of 3089 daily observations of Bitcoin market price were selected from July 16^{th}, 2010 to December 31^{st}, 2018.

The statistics of the logarithm return rate series of 100 simulated prices are plotted from Figures 3.15 to 3.18. The mean and variance of all the simulated series is concentrated within a narrow range. In addition, the skewness of all the simulated series is far from the skewness of normal

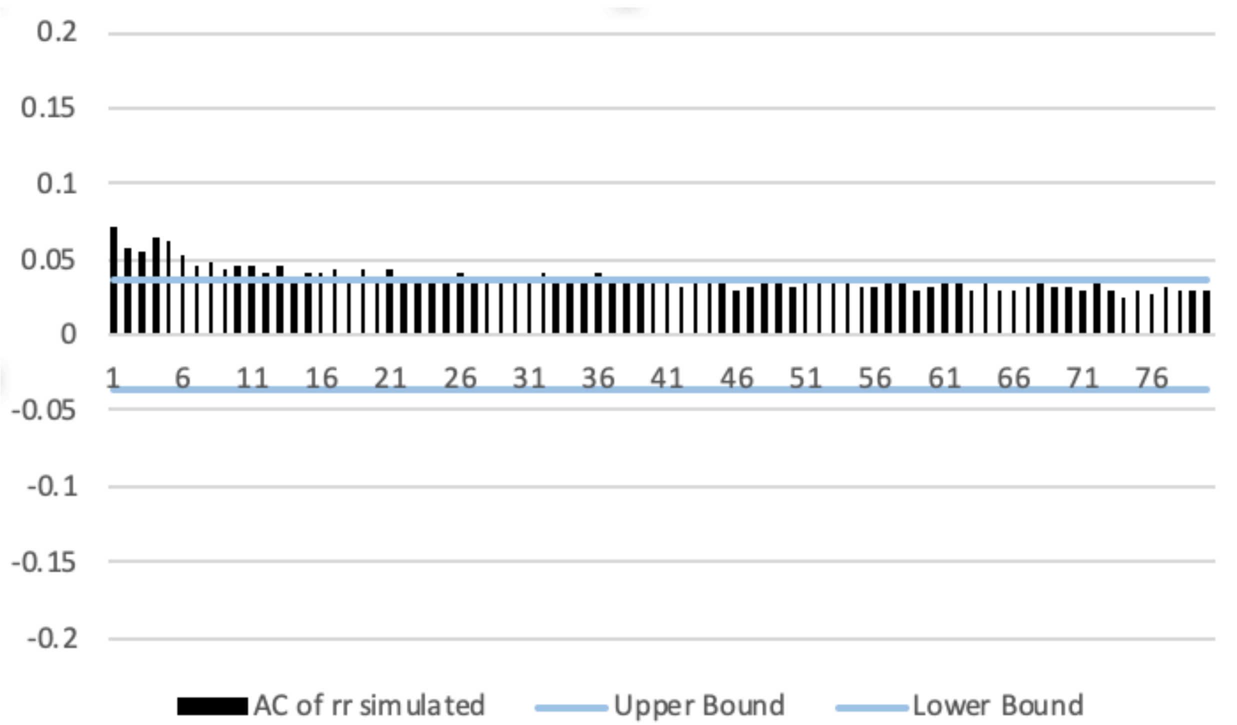


Figure 3.12 Auto-Correlation of Simulated Return Rate

distribution. Moreover, the kurtosis and the studentized range statistics (which is the range divided by the standard deviation), combined with the values of skewness, exhibit that characteristic fat-tailed behaviour appear in the simulated series, compared to a normal distribution. Furthermore, the Jarque-Bera statistics ($JB = n[\frac{Skewness^2}{6} + \frac{(Kurotosis-3)^2}{24}]$) follow Chi-square distribution with a degree of freedom of 2. Under 5% significance level, the critical value is 10.6. The smallest Jarque-Bera statistics of the simulated series is greater than 2,000, which is much greater than the critical value. Therefore, the simulated return rates are far different from a normal distribution.

Similarly, the statistics of the logarithm return rate series of historical Bitcoin market price are summarized in Table 3.2, and the average of the statistics of the simulated series are listed as well. The mean of the historical series is close to zero, which is similar to that of the simulated series. Additionally, all the variances of the simulated return rate are less than that of the historical data

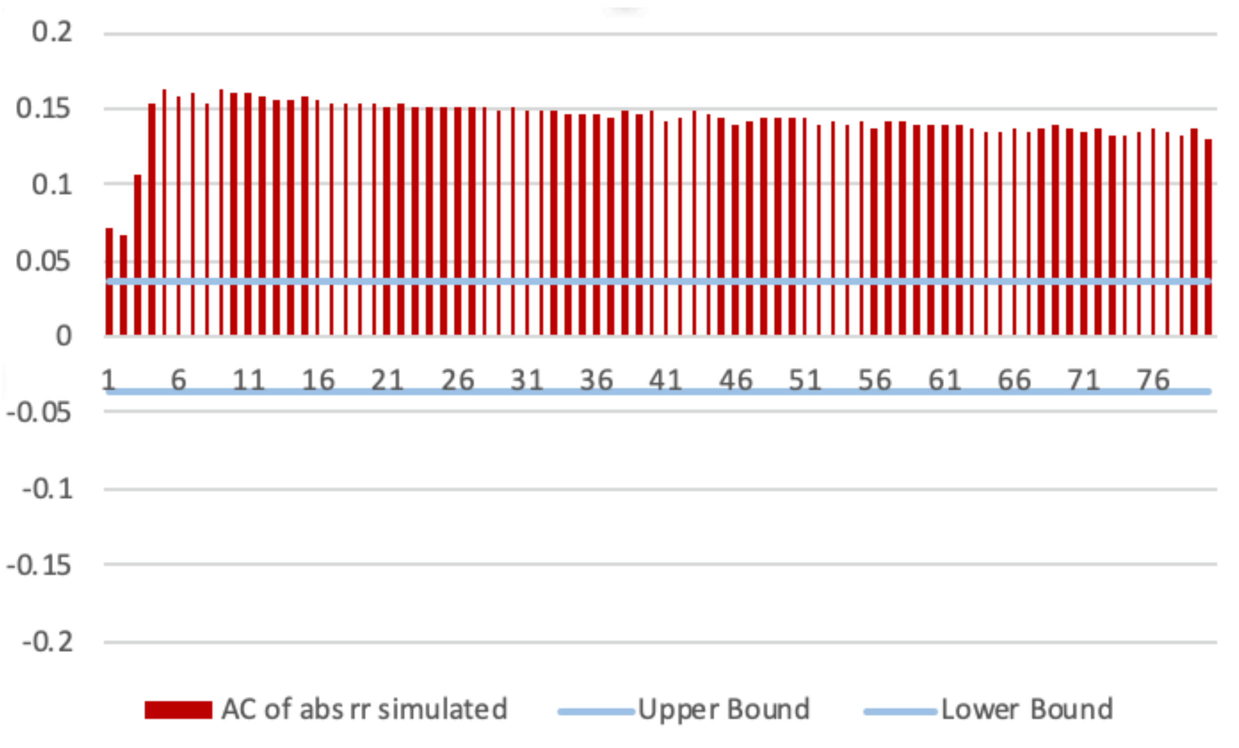


Figure 3.13 Auto-Correlation of Absolute Simulated Return Rate

which indicates the relative lower volatility appears in simulated data. Moreover, the skewness of the historical series is also deviated from the skewness of the normal distribution, particularly, the smallest skewness of the simulated series is greater than that of historical return rates. Furthermore, the kurtosis and the studentized range statistics of the historical series display the characteristic of fat-tailed behaviour as well. Finally, the Jarque-Bera statistics of the historical series is greater than the critical value, indicating the historical series are different from the normal distribution.

First of all, the stationary of the simulated and historical series are tested separately. Augmented Dickey-Fuller (ADF) tests are applied to check the stationary of the series. Consider two general AR(p) models:

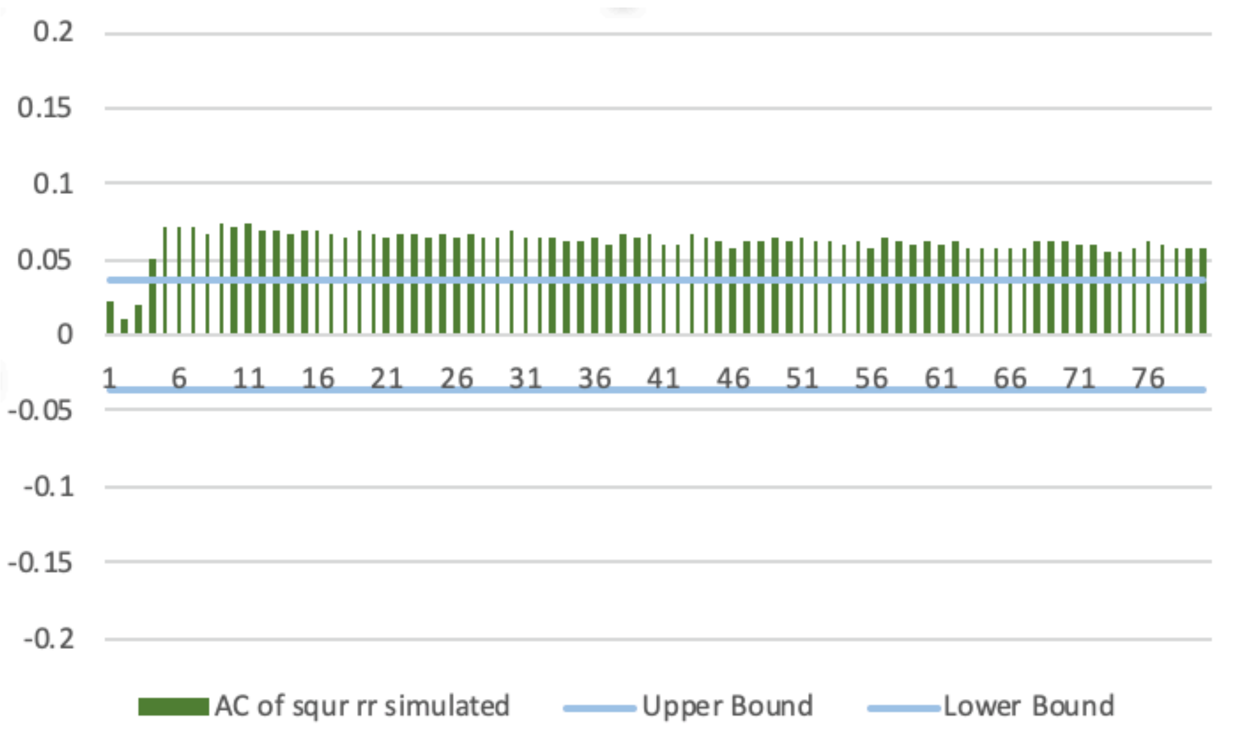
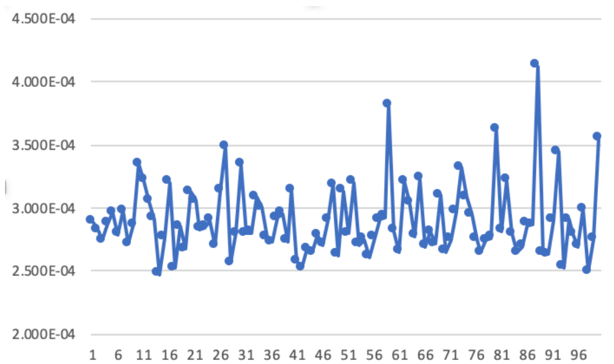


Figure 3.14 Auto-Correlation of Squared Simulated Return Rate

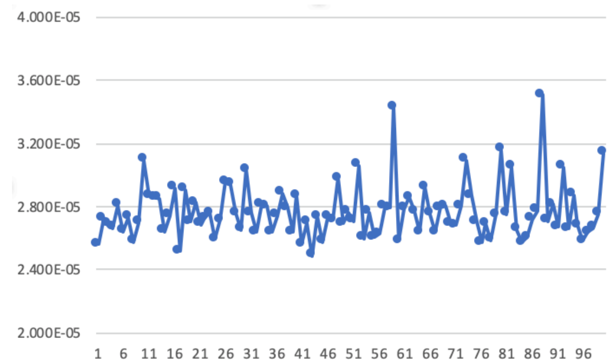
$$x_t = \beta x_{t-1} + \sum_{i=1}^{p-1} \phi_i \Delta x_{t-i} + e_t \tag{3.42}$$

$$x_t = c_t + \beta x_{t-1} + \sum_{i=1}^{p-1} \phi_i \Delta x_{t-i} + e_t \tag{3.43}$$

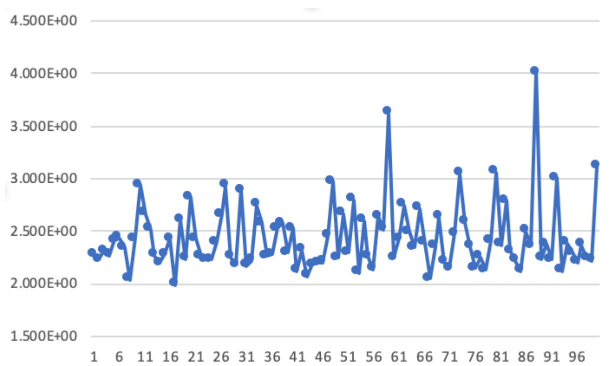
The null hypothesis of ADF test is H_0: β=1 and the alternative hypothesis is H_1: β<1. c_t is the deterministic function of time t which can be zero, a constant or in the form of ω_0+$\omega_1 t$. Additionally, Δx_j=x_j−x_{j-1}. The t-ratio of $\hat{\beta}-1$ is $\frac{\hat{\beta}-1}{\sigma(\hat{\beta})}$ which is the statistic of ADF tested. Here $\hat{\beta}$ is the least squared estimation of β and $\sigma(\hat{\beta})$ is the corresponding standard deviation.



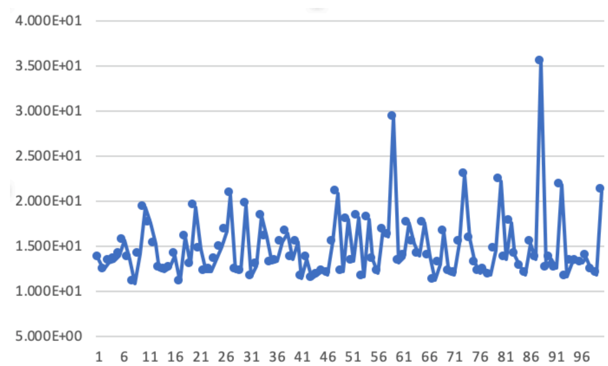
(a) Mean



(b) Variance



(c) Skewness

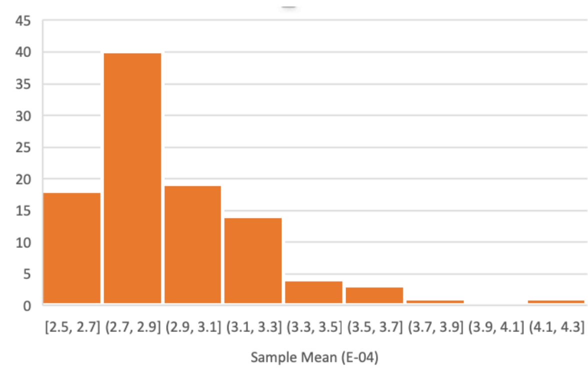


(d) Kurtosis

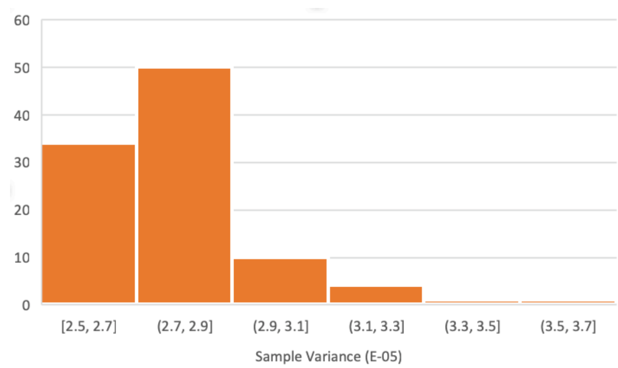
Figure 3.15 Statistics of Simulated Return Rate Series A1

Figure 3.19 shows the results of ADF test of the simulated return rate series. The results in both cases (intercept included/not included) are less than -8, which indicates that ADF statistics reject the null hypothesis at 99% significance level. All the simulated return rate series are stationary.

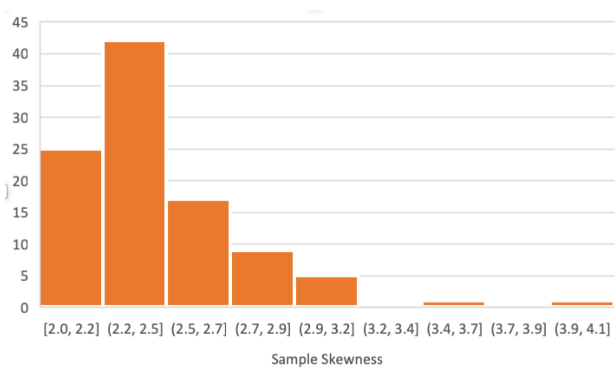
Similarly, for the historical return rate series, the p-values in both cases are less than 0.001 as displayed in Table 3.3. In additional, the average of ADF of the simulated series are also listed. Therefore, both the historical and simulated return rate series are stationary.



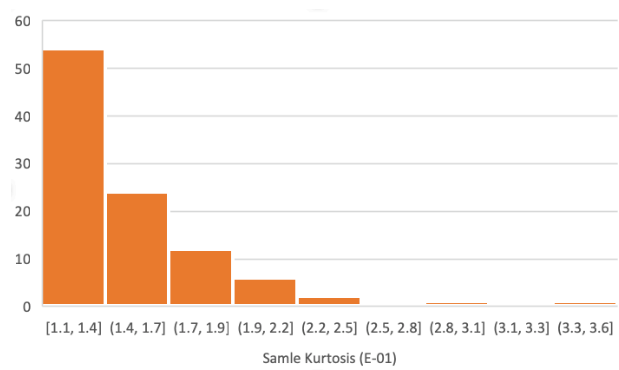
(a) Histogram of Mean



(b) Histogram of Variance



(c) Histogram of Skewness

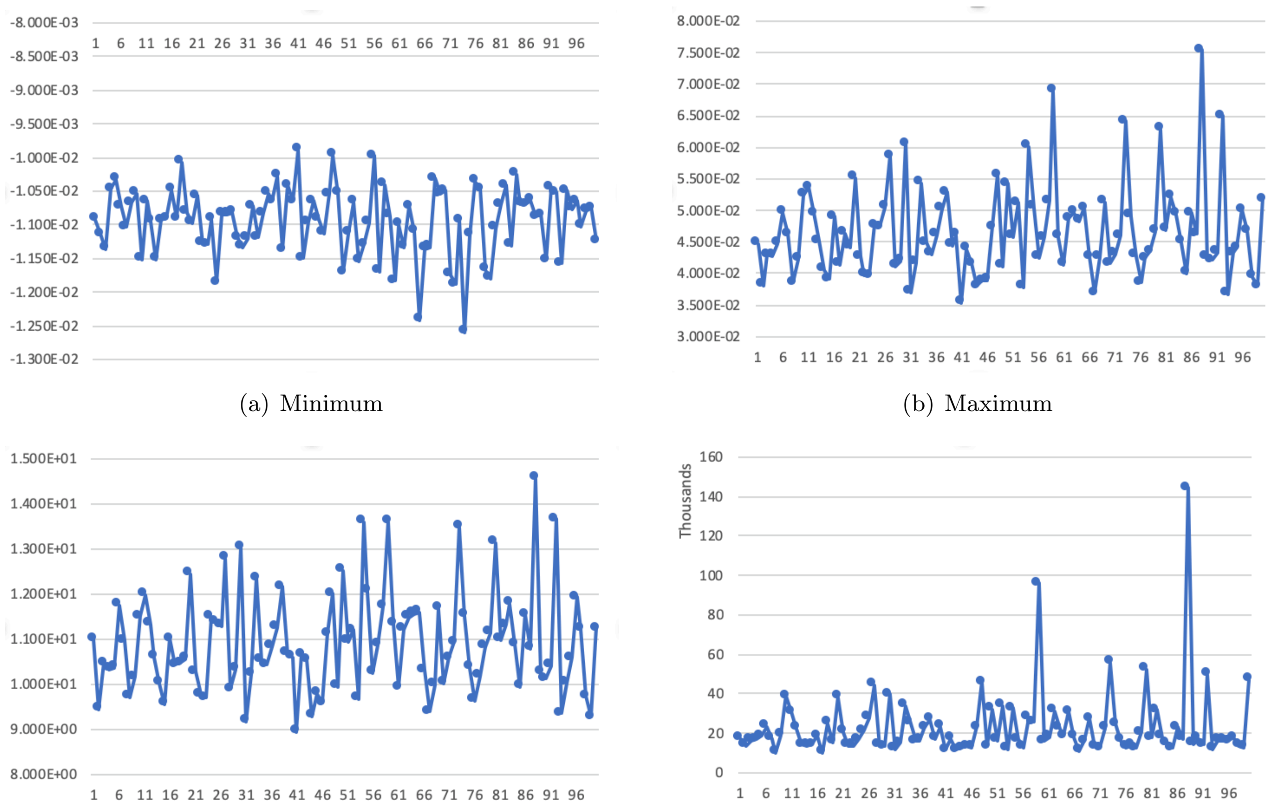


(d) Histogram of Kurtosis

Figure 3.16 Statistics of Simulated Return Rate Series A2

Figure 3.20 displays the historical return rate series. The historical return rate series fluctuate around zero. However, a few high volatile values are observed, which implies that some extreme events happened in Bitcoin history.

AR(p) model is applied to fit the stationary return rate series in order to capture its long-term trend. The lagging order is determined by ACF function. $p = 5$ is the lagging order for the both historical and simulated return rate series. Figure 3.21 displays the coefficients estimated for the simulated series. In addition, the estimation results of the historical series are summarized in Table 3.4. For the historical return rate series, the estimated parameters of items AR_2, AR_4, AR_5 and intercept are statistically significant.



(a) Minimum

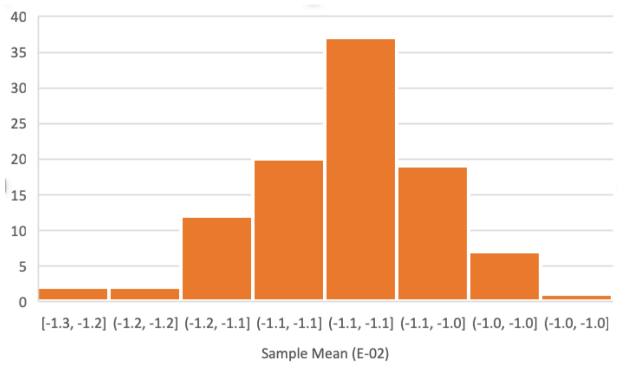
(b) Maximum

(c) Studentized Range

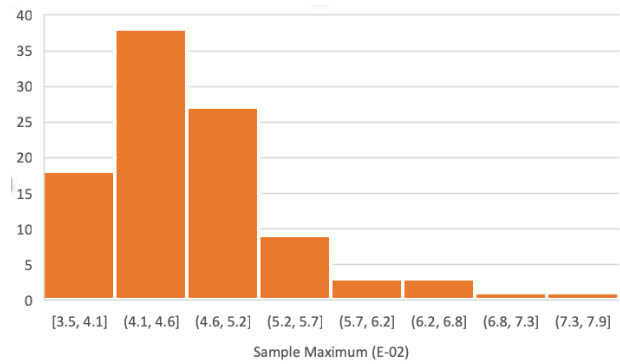
(d) Jarque-Bera

Figure 3.17 Statistics of Simulated Return Rate Series B1

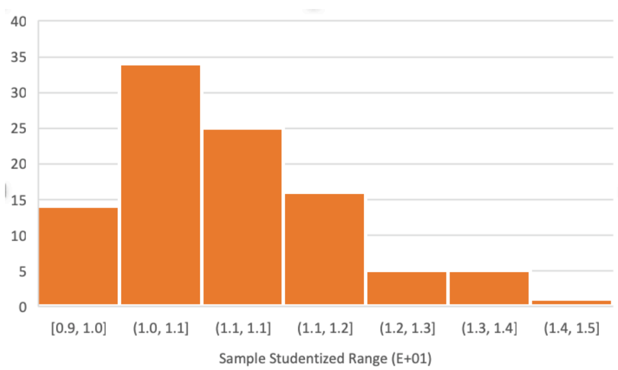
The deviation of Skewness and Kurtosis from that of normal distribution, the volatility cluster (high/low volatility followed by high/low volatility) are the stylized facts of financial time series. In order to investigate the volatility cluster, [83] proposed a test in which the null hypothesis is that the residual of fitted regression model is $i.i.d$, and the alternative hypothesis is that the residual follows auto-regressive conditional heteroskedasticity ($ARCH(q)$) process. In other words, if the return rate series follows $AR(p)$ process and its innovation is defined as a_t, then the variance in a_t^2 will be purely random if the return rates are homoscedastic. However, if ARCH effects are present, the large value of a_t^2 can be predicted from the previous large value of squared residuals.



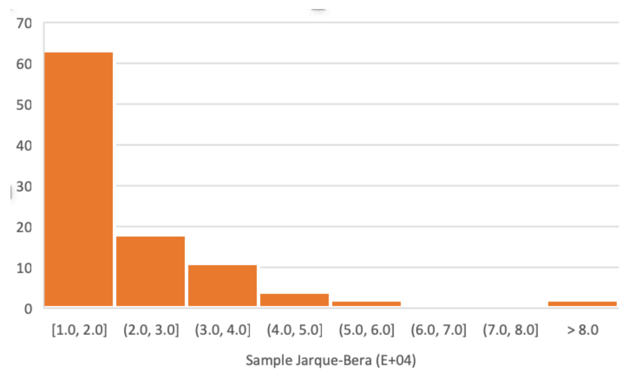
(a) Histogram of Minimum



(b) Histogram of Maximum



(c) Histogram Studentized Range



(d) Histogram of Jarque-Bera

Figure 3.18 Statistics of Simulated Return Rate Series B2

In general, Ljung-Box test is applied to investigate the fitness of AR model and the ARCH effect. With $a_t = r_t - \mu_t$ denoted as the residual of fitted mean equation, Ljung-Box statistics $Q(m)$ is applied to the series of residual $\{a_t\}$ and that of squared residual $\{a_t^2\}$. The null hypothesis is that for the series, the first m lags of ACF are zero and no ARCH effect exists. If $Q(m) > \chi^2_\alpha$, the null hypothesis is rejected and the ARCH effect exists in the series. Here, χ^2_α denotes the $100(1-\alpha)^{th}$ percentile of a Chi-squared distribution with m degrees of freedom:

$$Q(m) = T(T+2)\sum_{l=1}^{m} \frac{\hat{\varrho}_l^2}{T-l} \tag{3.44}$$

Table 3.2 Statistics Summary of Return Rate Series

Statistic	Historical	Simulated
Span	3088	3099
Mean	1.585e-03	2.904e-04
Variance	8.837e-04	2.769e-05
Skewness	2.935	2.432
Kurtosis	91.127	14.788
Min	-3.686e-01	-1.091e-02
Max	6.403e-01	4.650e-02
Stud.Range	33.940	10.906
Jarque-Bera	1074300	228670
p-Value	$< 2.2e$-16	$< 2.2e$-16

Note: The statistics of simulated series are the average of statistics for each simulated series.

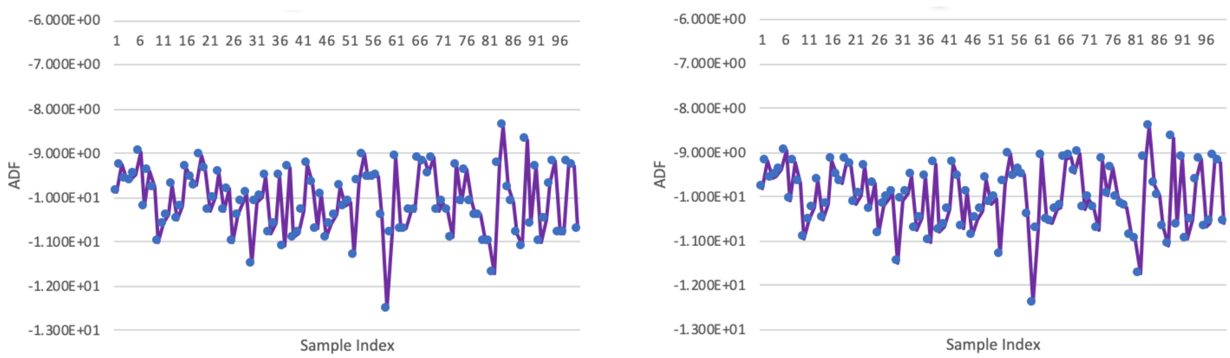
Table 3.3 Summary of ADF of Return Rate Series

Variable	Lag Order	Dickey-Fuller	p-Value
Historical Intercept included	30	-8.434	<0.001
Historical Intercept not included	30	-8.434	<0.001
Simulated Intercept included	30	-10.011	<0.001
Simulated Intercept not included	30	-10.011	<0.001

Note: The statistics of simulated series are the average of statistics for each simulated series.

where $\hat{\varrho}_l$ is the auto-correlation coefficient.

The p-values of Ljung-Box test with the different lagging orders of the estimated residuals and squared residuals for the simulated return rate series are displayed in Figures 3.22 and 3.23. For the simulated residuals, all p-values of lagging order 1 and 2 are greater than 0.1. Additionally, nearly half of the p-values of lagging order 5 are also greater than the critical value. Moreover, most of the p-values of lagging order 10 and 20 are less than 0.1. Therefore, the simulated AR(5) model can capture AC effect of the simulated return rates till lagging 5. Besides, in Figure 3.23, for the squared residuals, all the p-values of Ljung-Box test are less than 0.01, which indicates strong ARCH effect exists in the simulated series till lagging 5 at least.



(a) ADF without intercept

(b) ADF with intercept

Figure 3.19 ADF Test Result of Simulated Return Rate Series

Table 3.4 Summary of AR(5) Estimation of Return Rate Series

Coefficient	AR(1)	AR(2)	AR(3)	AR(4)	AR(5)	Intercept
Historical	2.407*e*-02	-1.592*e*-01	2.292*e*-02	8.511*e*-02	9.682*e*-02	1.586*e*-03
	(1.813*e*-02)	(1.804*e*-02)***	(1.826*e*-02)	(1.804*e*-02)*	(1.828*e*-02)*	(5.616*e*-04)*
Simulated	1.187*e*-01	1.107*e*-03	1.073*e*-01	1.238*e*-01	1.224*e*-01	8.816*e*-05
	(2.689*e*-02)	(2.818*e*-02)	(2.937*e*-02)	(2.560*e*-02)	(2.502*e*-02)	(8.408*e*-05)

Note: The statistics of simulated series are the average of statistics for each simulated series.

On the other side, for the historical series, the results in Table 3.5 show that Ljung-Box statistics of the residual sequence in all lagging orders are significant, which indicates that no significant AC pattern exists in the residual series. Therefore, most of the AC effect has been captured well by the fitted AR(5) model. Meanwhile, all the Ljung-Box statistics of the squared residual are not significant with p-value is <0.01, indicating the strong ARCH effect exists in the residual series.

The AC of the return rates, the absolute return rates and the squared return rates of the historical series are displayed from Figure 3.24 to 3.26 respectively. The market price shows volatility clustering, which is characterized by the significant AC pattern of the return rates. The coefficients of AC are statistically significant till lagging 7 for the return rates of historical market price, lagging 34 for the absolute return rate series and are statistically significant till lag 6 for the squared return

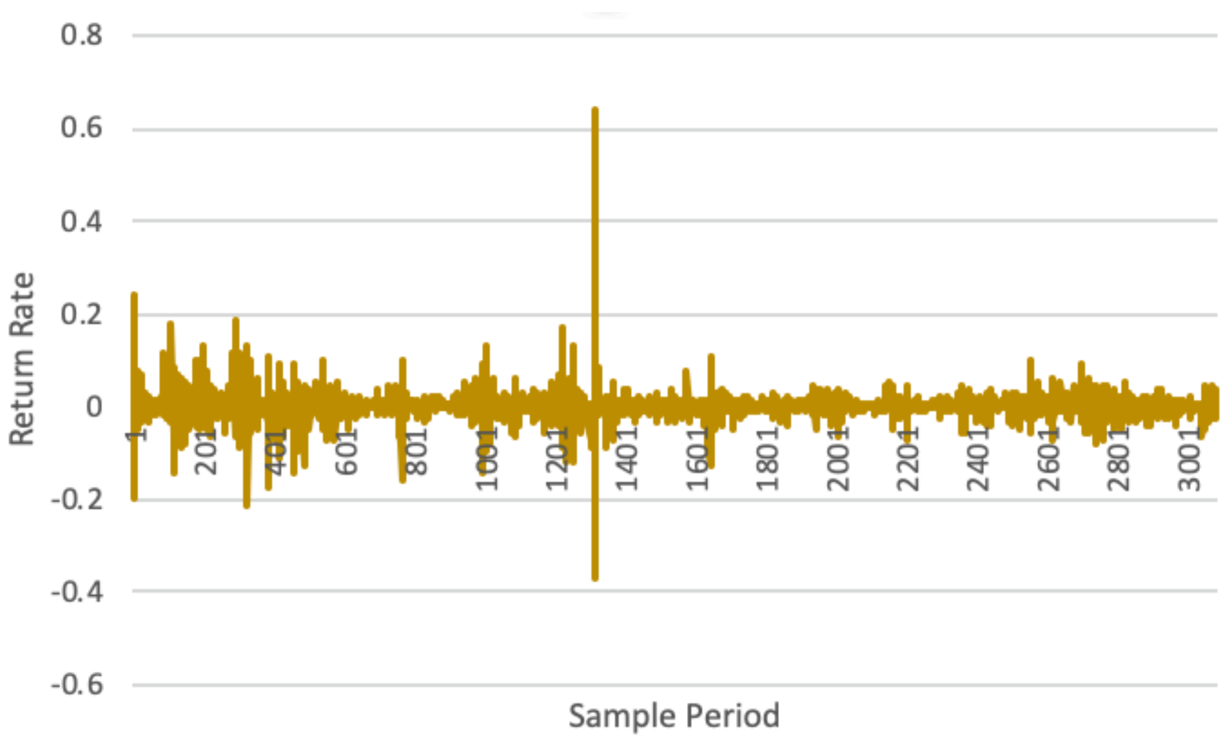


Figure 3.20 Return Rate of Historical Market Price Series

rates, which implies the long-term dependence for the return rates. Compared to the AC patterns of simulated series in Figures 3.12-3.14, it can be concluded that the simulated return rate series have the similar statistical properties to that of the historical market price series.

In order to explore more details of the long-term memory feature of the sequence, the combination of ADF and Kwiatkowski-Phillips-Schmidt-Shin (KPSS) ([84]) test for the series of squared residuals are performed. Specifically, if the sequence rejects ADF and KPSS test simultaneously, then it is neither a $I(1)$ nor a $I(0)$ sequence, but is a fractional-differentiated process which has a long-term memory feature. More details of KPSS test are illustrated as follows. Consider time series x_t for $t = 1, 2, ...T$. $x_t = a_0 + a_1 t + z_t$, where $z_t = u_t + \varepsilon_t$, and u_t as random walk which satisfies:

$$u_t = u_{t-1} + \nu_t,\ u_0 = 0,\ \nu_t \sim i.i.d\ (0, \nu_t^2);\ \nu_t^2 < \infty \tag{3.45}$$

Table 3.5 Summary of Ljung-Box Test of Estimated Residuals Series

	Lag 1	Lag 2	Lag 5	Lag 10	Lag 20
Historical a_t χ^2	1.191e-02	1.524e-01	2.579e-01	6.610	17.232
	(9.131e-01)	(9.267e-01)	(9.984e-01)	(7.617e-01)	(6.379e-01)
Historical a_t^2 χ^2	8.283	113.76	333.980	592.930	595.980
	(4.002e-03)	($<$ 2.200e-16)	($<$ 2.200e-16)	($<$ 2.200e-16)	($<$ 2.200e-16)
Simulated a_t χ^2	7.618e-01	3.545	13.209	54.807	93.792
	(4.347e-01)	(2.196e-01)	(5.246e-02)	(2.917e-05)	(4.370e-07)
Simulated a_t^2 χ^2	58.867	96.064	200.851	424.722	728.454
	($<$ 2.000e-08)	($<$ 2.000e-08)	($<$ 2.000e-08)	($<$ 2.000e-08)	($<$ 2.000e-08)

Note: The numbers in parentheses are p-values.
The statistics of simulated series are the average of statistics for each simulated series.

The null hypothesis of KPSS test is $\nu_t^2 = 0$, *i.e.*, series z_t is of the short-term memory. The statistic is:

$$\xi_t = T^{-2} \sum_{t=1}^{T} \frac{S_t^2}{\sigma_q^2(\tau)} \tag{3.46}$$

where $S_t = \sum_{j=1}^{t}(x_j - \bar{x})$, $t = 1, 2, ..., T$; and

$$\sigma_q^2(\tau) = \frac{1}{\tau}\sum_{j=1}^{\tau}(x_j - \bar{x}_\tau)^2 + \frac{2}{\tau}\sum_{j=1}^{q}\left(1 - \frac{j}{q+1}\right)\left[\sum_{i=j+1}^{\tau}(x_i - \bar{x}_\tau)(x_{i-j} - \bar{x}_\tau)\right] \tag{3.47}$$

Here, $\bar{x}_\tau = \frac{1}{\tau}\sum_{i=1}^{\tau} x_i$.

Figure 3.27 shows the ADF & KPSS test results of the simulated series. Given 5% significance level, the null hypothesis is rejected in both ADF and KPSS test, which indicates the squared residual series of fitted AR (5) for all simulated return rate series are fractional differential sequences, namely, the return rate series have the characteristics of long memory. The current information will have an impact on the future prices persistently, and the impact decays over time. Additionally, the rate of decay is slower than that of the short-term memory sequence.

For the historical series, the ADF and KPSS test results in Table 3.6 suggest that the Bitcoin historical market price also has the feature of long-term memory. The price is sensitive to the historical information. The speculators in the market indeed follow the previous prices to guide their current bids.

Table 3.6 Summary of ADF & KPSS of Squared Residual Series

Variable	Lag Order	Statistic	p-Value
ADF Test Historical Series	20	-7.950	<0.01
KPSS Test Historical Series	15	5.420e-01	3.73e-01
ADF Test Simulated Series	20	-8.452	<0.001
KPSS Test Simulated Series	15	2.071	<0.01

Family of ARCH model is applied to capture the ARCH effect. The generalized ARCH (GARCH) model, which is mostly used, is provided by [85] followed by ARCH model designed in [83]. The specification of GARCH model is:

$$y_t = \mu_t + a_t \tag{3.48}$$

where $\{y_t\}$ is a stationary series, $\{\mu_t\}$ is mean equation and $\{a_t\}$ is the residual (innovation) series.

$$a_t = \sigma_t \epsilon_t, \quad \sigma_t^2 = \alpha_0 + \alpha(L) a_t^2 + \beta(L) \sigma_t^2 \tag{3.49}$$

where ϵ_t is an $i.i.d$ pure random process with mean of 0 and variance of 1. $\alpha(L)$ and $\beta(L)$ can be written as: $\alpha(L) = \sum_{i=1}^{p} \alpha_i L^i$ and $\beta(L) = \sum_{j=1}^{q} \beta_j L^j$ with $\alpha_0 > 0$, $\alpha_i \geq 0$, $\beta_j \geq 0$ and $\sum_{i=1}^{max(p,\ q)} (\alpha_i + \beta_j) < 1$. In addition, define $v_t = a_t^2 - \sigma_t^2$, then $\{v_t\}$ is a martingale difference series. GARCH (1,1) model defined as equation (3.50), which is frequently applied to the financial time series data, is used to capture the volatility of return rate series.

$$(1 - \alpha_1 L - \beta_1 L)a_t^2 = \alpha_0 + (1 - \beta_1 L)v_t \tag{3.50}$$

Figure 3.28 plots the GARCH (1,1) estimated parameters for the simulated series. The estimated α_1 falls into the interval [0.04, 0.07] while the estimated β_1 locates within the interval [0.89, 0.92]. Hence, the impact of the recent innovations is small (small α_1), while the coefficients of variance reflect the characteristics of strong persistence (large β_1). In particular, the value of $\alpha_1 + \beta_1$ is quite close to one, which implies that the GARCH model is not most applicable, due to the fact that GARCH model describes the process in which shocks to the conditional variance decay exponentially. Table 3.7 lists the GARCH estimation of the historical series, which has the similar characteristics of the simulated series.

Table 3.7 Summary of GARCH(1,1) Parameters Estimation Results

Parameter	μ	α_0	α_1	β_1
Historical	1.048e-05	8.655e-04	2.381e-01	7.032e-01
	(1.515e-06)***	(12.554e-04)**	(1.864e-02)***	(1.246e-02)***
Simulated	7.258e-06	9.053e-08	4.780e-02	9.020e-01
	(6.824e-07)	(6.015e-08)	(4.098e-03)	(3.714e-03)

Note: Robust standard errors in parentheses *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.
The statistics of simulated series are the average of statistics for each simulated series.

Integrated GARCH (IGARCH) model describes the process in which the shocks to conditional variance persist indefinitely. Moreover, [86] proposed Fractional-Integrated GARCH (FIGARCH) model based on GARCH, in which the shock to the conditional variance decays at a slow hyperbolic rate. This model is used to capture the long-term memory characteristics. The specification of FIGARCH$(1, d, 1)$ model is defined as:

$$(1 - \phi_1 L)(1 - L)^d a_t^2 = \alpha_0 + (1 - \beta_1 L)v_t \tag{3.51}$$

where $0 < d < 1$. α_0, ϕ_1, β_1, d are the four parameters for estimation. It is notable that equation (3.51) represents GARCH when $d = 0$ and represents IGARCH when $d = 1$.

Figures 3.29 and 3.30 describe the estimated FIGARCH (1,1) parameters for the simulated series. The estimated $\phi_1 + \beta_1$ falls within the interval [0.741, 0.801], which indicates that the impact is persistent to some extent and decays slowly. In addition, the range of key coefficient d estimated is [0.882, 0.999], which implies that the estimated FIGARCH model is more close to IGARCH model, compared with GARCH model.

Similarly, in table 3.8, the estimated coefficient d for historical series equals 0.885. Current volatility of Bitcoin price will have a long-term impact on volatility in future. However, $\phi_1 + \beta_1$=0.743<1, which indicates that the impact is also persistent. The estimation results of historical and simulated return rate series are summarized in Table 3.8 below.

Table 3.8 Summary of FIGARCH Estimation

Parameter	ϕ_1	α_0	β_1	d
Historical	$4.241e^{-2}$	$1.863e^{-4}$	$7.516e^{-1}$	$8.855e^{-1}$
	$(3.852e^{-2})$	$(2.093e^{-2})$	$(1.596e^{-2})$**	$(7.303e^{-3})$***
Simulated	$1.795e^{-2}$	$8.076e^{-7}$	$7.462e^{-1}$	$9.324e^{-1}$
	$(3.224e^{-2})$	$(1.576e^{-6})$	$(6.360e^{-3})$	$(3.721e^{-2})$

Note: Robust standard errors in parentheses *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

The statistics of simulated series are the average of statistics for each simulated series.

Overall, the significant ARCH effect which is of long-term memory for Bitcoin price series has been confirmed in the empirical analysis. The major events in Bitcoin's history, such as Bitcoin mining disaster in 2013, price skyrocket rising in 2017 have brought high volatility to Bitcoin market price. In addition, the market price is very sensitive to its previous values. The information revealed today will be referred for a long period in future, which is the evidence of trend following feature of Bitcoin speculators. Moreover, the price series generated by numerical simulations have very similar statistical properties to that of historical price series. In other words, the phenomenons such as long term memory and path dependence have been captured well.

3.6 Conclusion

This paper provides an economic framework for analyzing the characteristics of Bitcoin market price. Bitcoin system is a new technology designed by [1], which serves as a peer-to-peer payment network. Compared with the traditional digital payment methods, un-revocability and anonymity of wealth transfer are the advantages of Bitcoin. Meanwhile, the transaction congestion caused by low-efficient validation process is its disadvantage. During most of the time after its debut, the market cap of Bitcoin has not been much different from that of other mainstream payment systems. However, the market cap has sharply increased and declined for several times. In this paper, an economic model is developed to determine the Bitcoin market price.

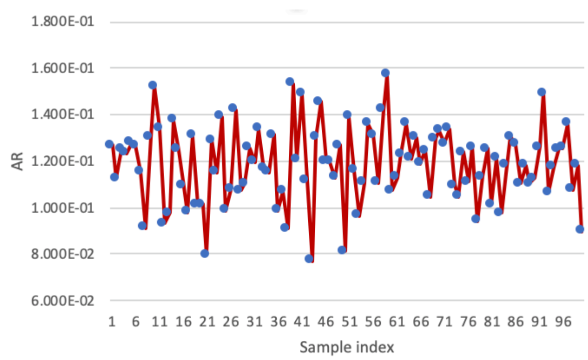
The participants in Bitcoin trading market are categorized as fundamentalists and speculators, and the interactions between these two types of participants determine the market price of Bitcoin at each period. On one side, the fundamentalists have the ability to collect the information about intrinsic value of Bitcoin system and use this information to predict the market price of Bitcoin. On the other side, the speculators heuristically explore the information of the previous prices and follow the strategy of social imitation to make their investment decisions.

Compared with the previous literature on financial market bubbles, there are two essential innovations in this model. Firstly, instead of being set as exogenous, the fundamental value of Bitcoin system is considered as a variable, which depends on the maturity of technology and level of public acceptance. The second one is that a non-linear function of overall speculator's opinion based on price deviation is constructed, which reflects the speculators' sentiment variety on price change. These two innovations lead to the new characteristics of market price evolution.

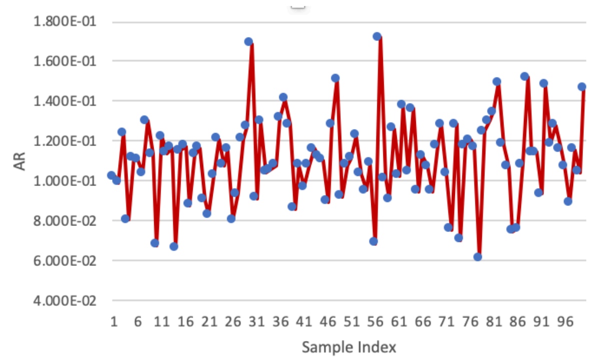
Econometric tools are necessary to perform statistical analysis. Statistical analysis on both Bitcoin prices simulated from the theoretical model and historical price of Bitcoin is performed. Our model reproduces some typical facts of speculative behaviour in financial markets, such as the fat tail feature of daily return series and significant ARCH effect (volatility clustering). Besides this, the empirical analysis has also confirmed that the return rate series has characteristics of long-term memory. Current price depends on its previous prices, which has a persistent impact on the future prices as well. Speculators in the trading market perform their investment referring to the previous market prices as designed in the theoretical model.

Bitcoin is not a scam. Similar to other nascent technologies, Bitcoin system has its own specific usage and has some potential applications which may bring innovations to the current business pattern as well. Therefore, Bitcoin system has the intrinsic value, while it still requires a lot of improvement. However, the innovative technology attracts the speculators in the financial market, and is recognized as a speculating tool. The market price of Bitcoin dramatically increases and more speculators are attracted. The investing strategy of speculators leads to the soaring of prices and price bubble eventually. Once market sentiment begins to calm down, the bubble will burst. This is similar to the previous NASDAQ technology bubble and could happen again in future on Bitcoin or other underlying assets.

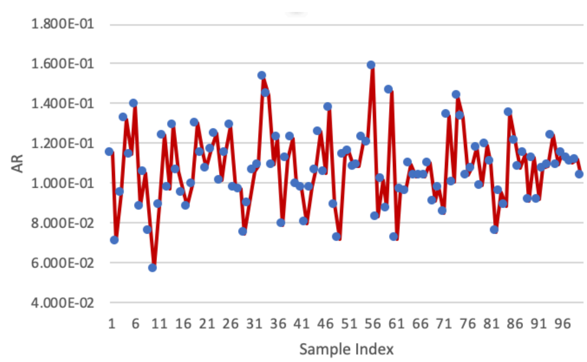
Some further investigations and extensions of this research are of great importance. First, our model framework follows the classical literature in behavioural analysis of agents in traditional security trading market. In analysis of traditional financial market, exogenous wealth does not influence investment decisions. However, this setting may not be appropriate for Bitcoin market. Since Bitcoin can be divided infinitely, the impact of wealth from small transactions on the speculative decision, which accounts for a large share of market transactions, should not be ignored. Second, although the simulated price series can capture the statistical properties of the historical price series, its volatility is lower than that of historical price. The designed function of the overall speculators' opinion index is a simple quadratic nonlinear function of the deviation between current price and historical mean. Finally, the fundamental value in the model is mostly determined by the public acceptance, while the usage such as remittance can also be incorporated to determine the fundamental value of Bitcoin system.



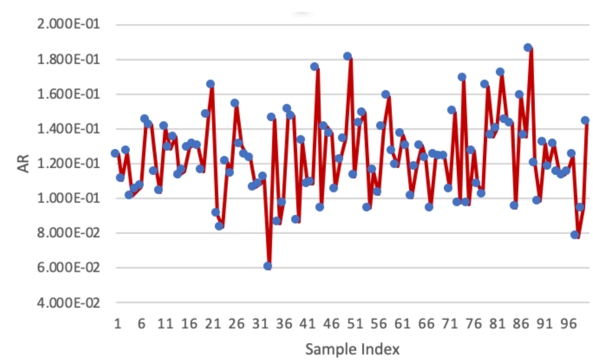
(a) AR_1 Coefficient



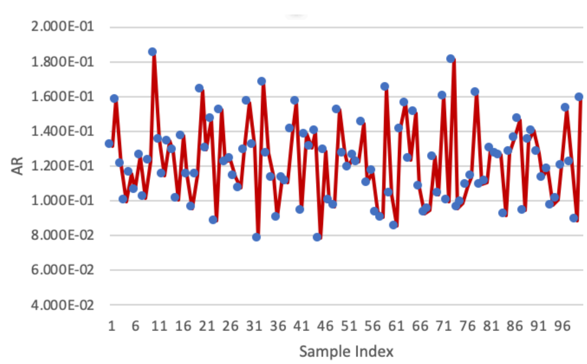
(b) AR_2 Coefficient



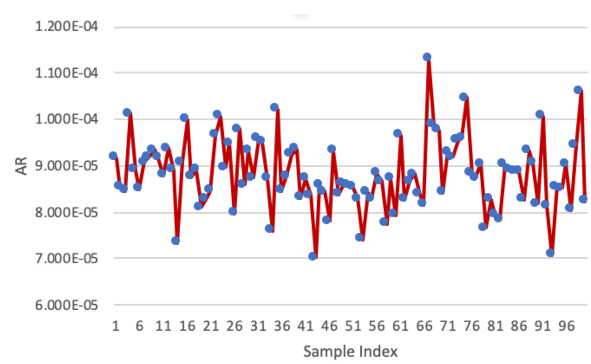
(c) AR_3 Coefficient



(d) AR_4 Coefficient

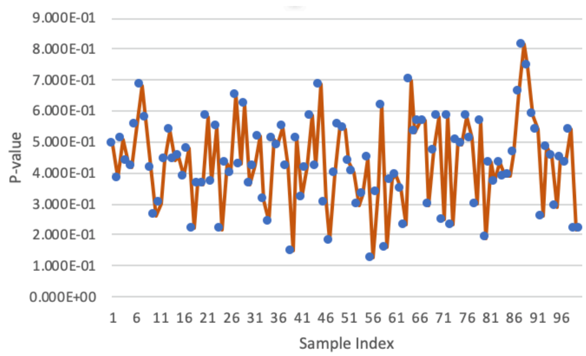


(e) AR_5 Coefficient

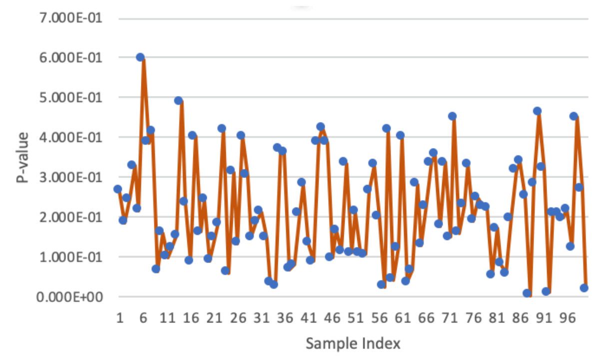


(f) *Intercept*

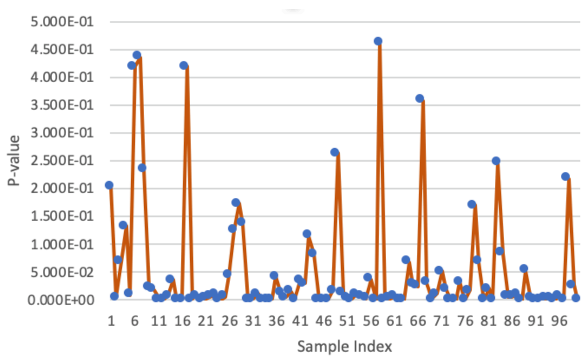
Figure 3.21 AR Coefficients of the Simulated Return Rate Series



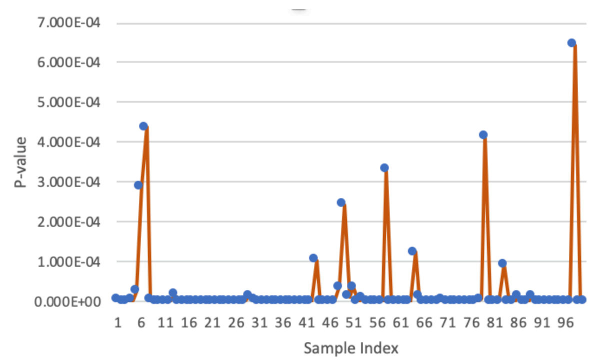
(a) Lagging Order 1



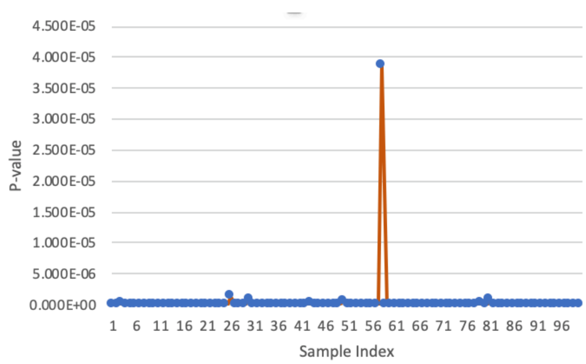
(b) Lagging Order 2



(c) Lagging Order 5

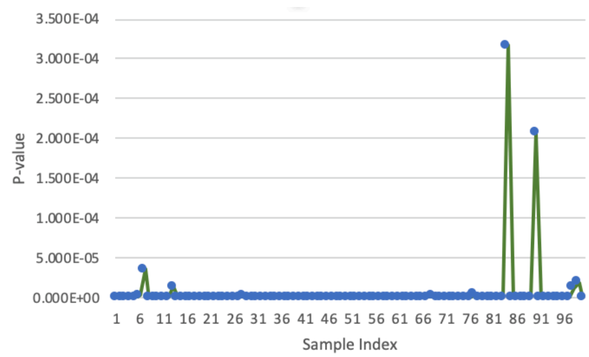


(d) Lagging Order 10

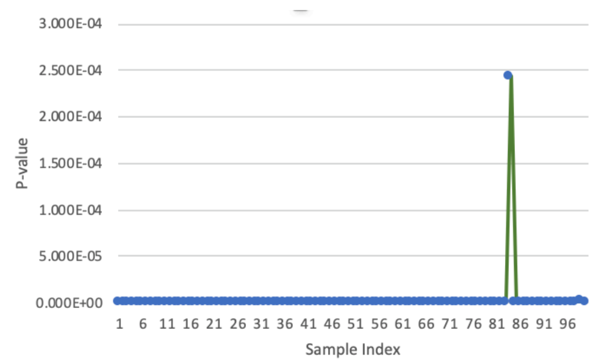


(e) Lagging Order 20

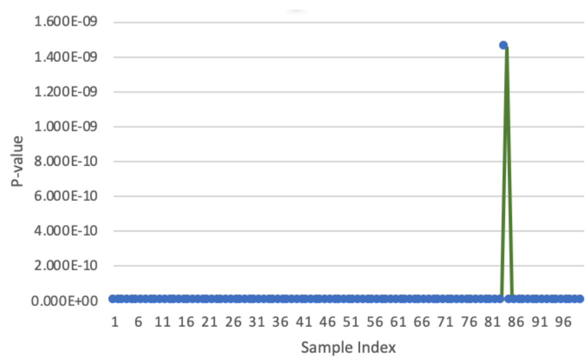
Figure 3.22 Ljung-Box Test p-Value of Estimated Residuals of the Simulated Return Rate Series



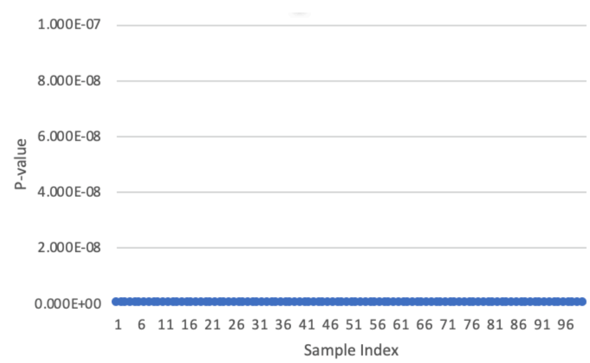
(a) Lagging Order 1



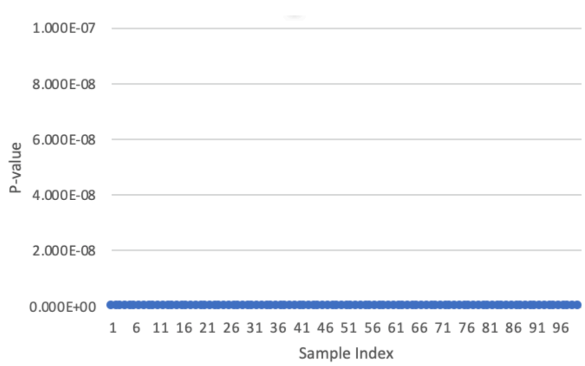
(b) Lagging Order 2



(c) Lagging Order 5



(d) Lagging Order 10



(e) Lagging Order 20

Figure 3.23 Ljung-Box Test p-Value of Estimated Squared Residuals of the Simulated Return Rate Series

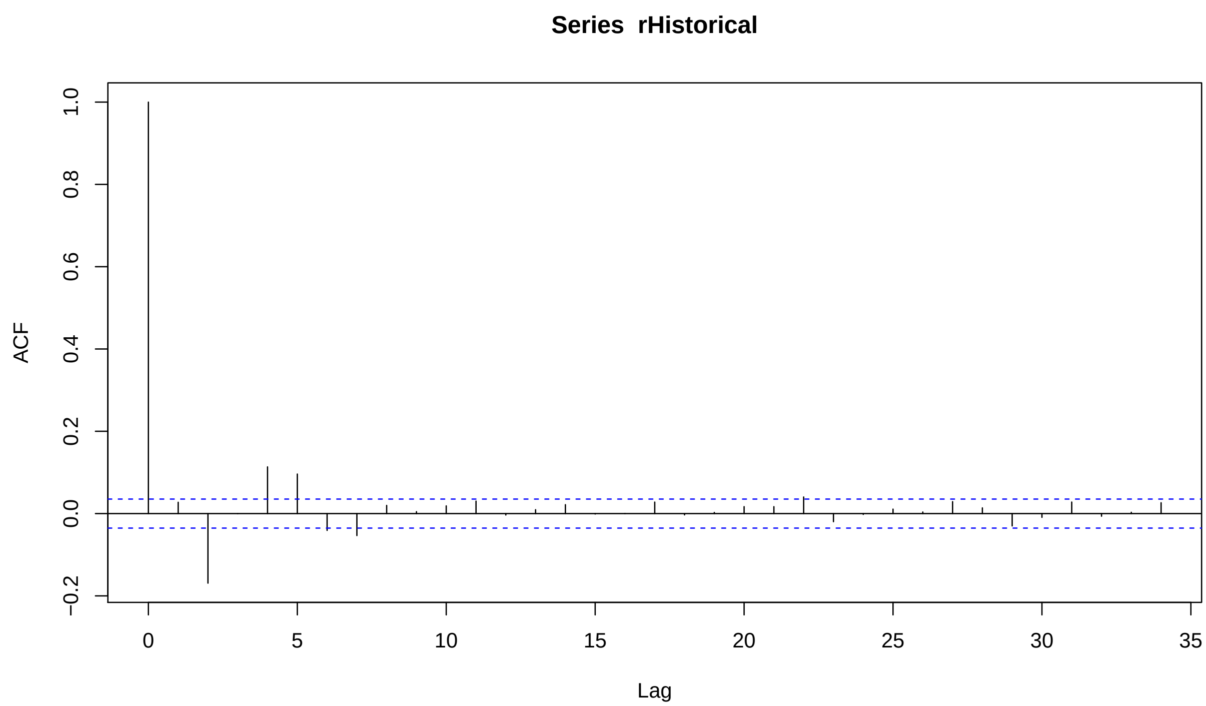


Figure 3.24 Auto-Correlation of Historical Return Rate

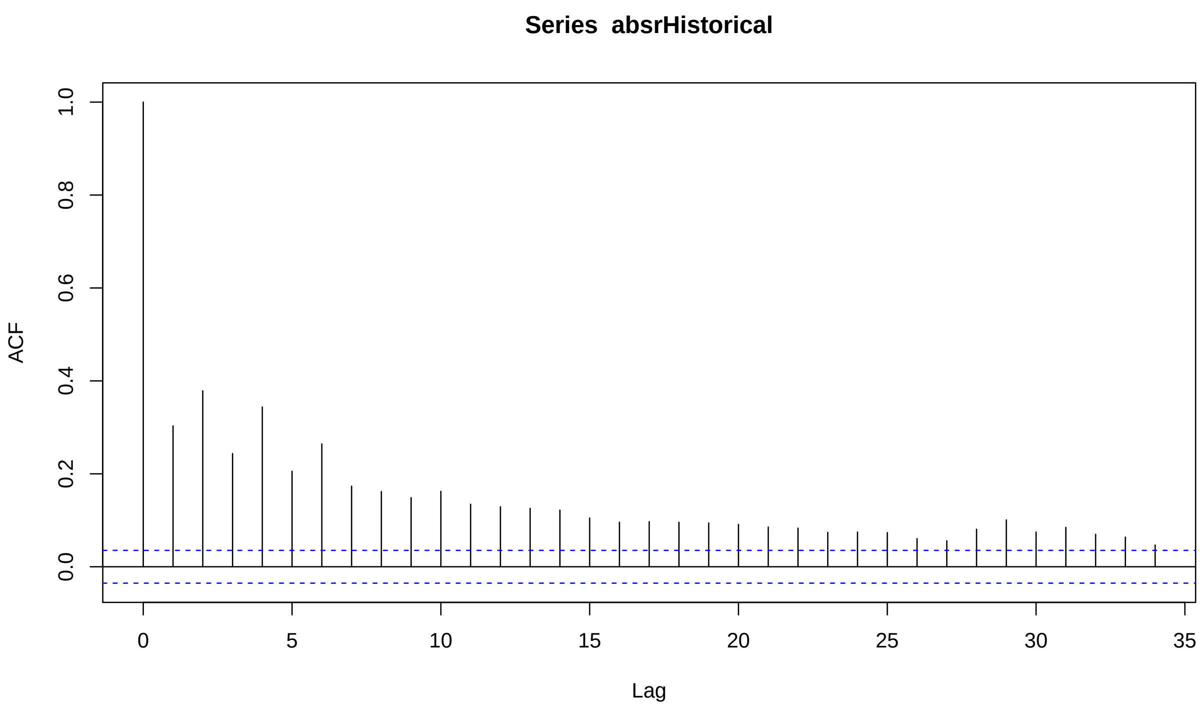


Figure 3.25 Auto-Correlation of Absolute Historical Return Rate

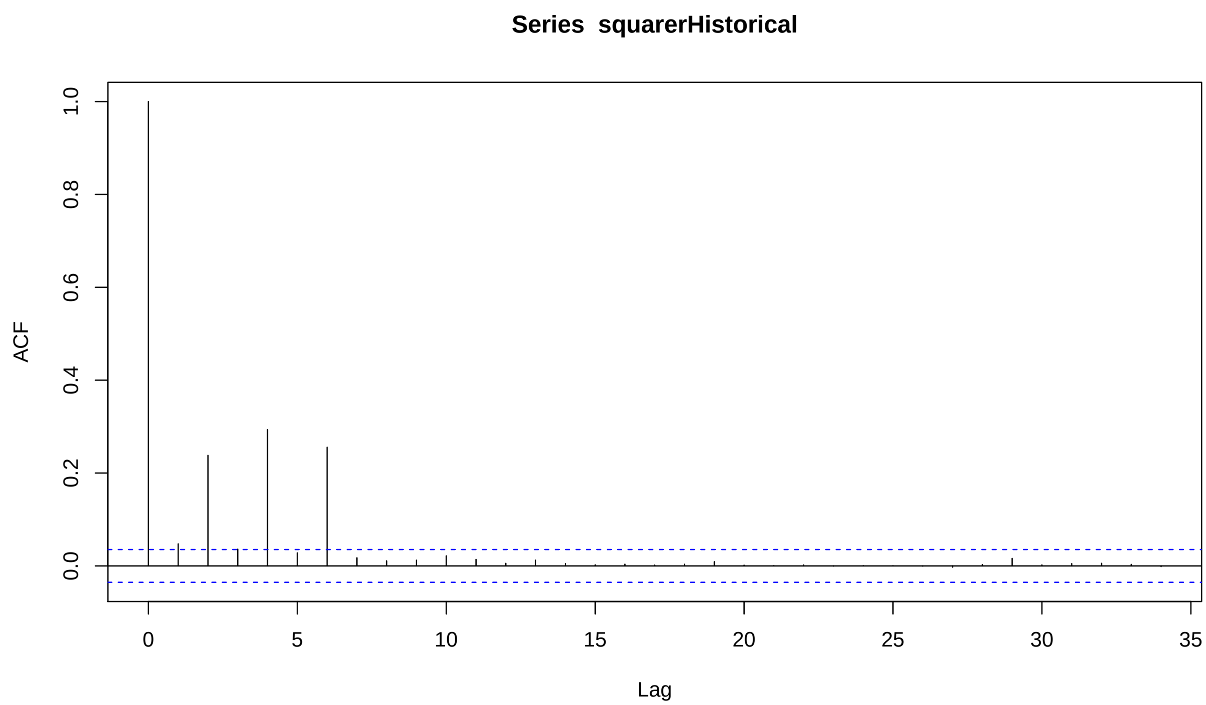
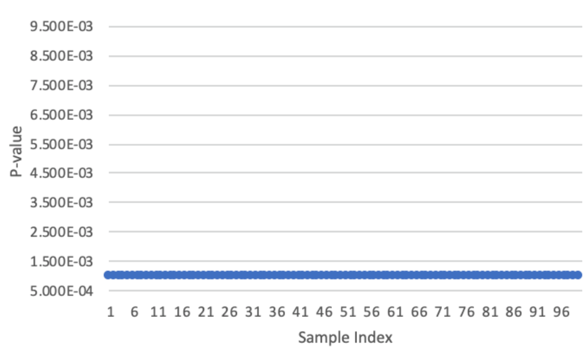
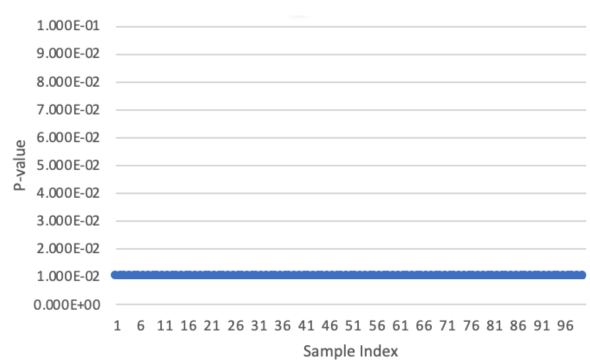


Figure 3.26 Auto-Correlation of Squared Historical Return Rate

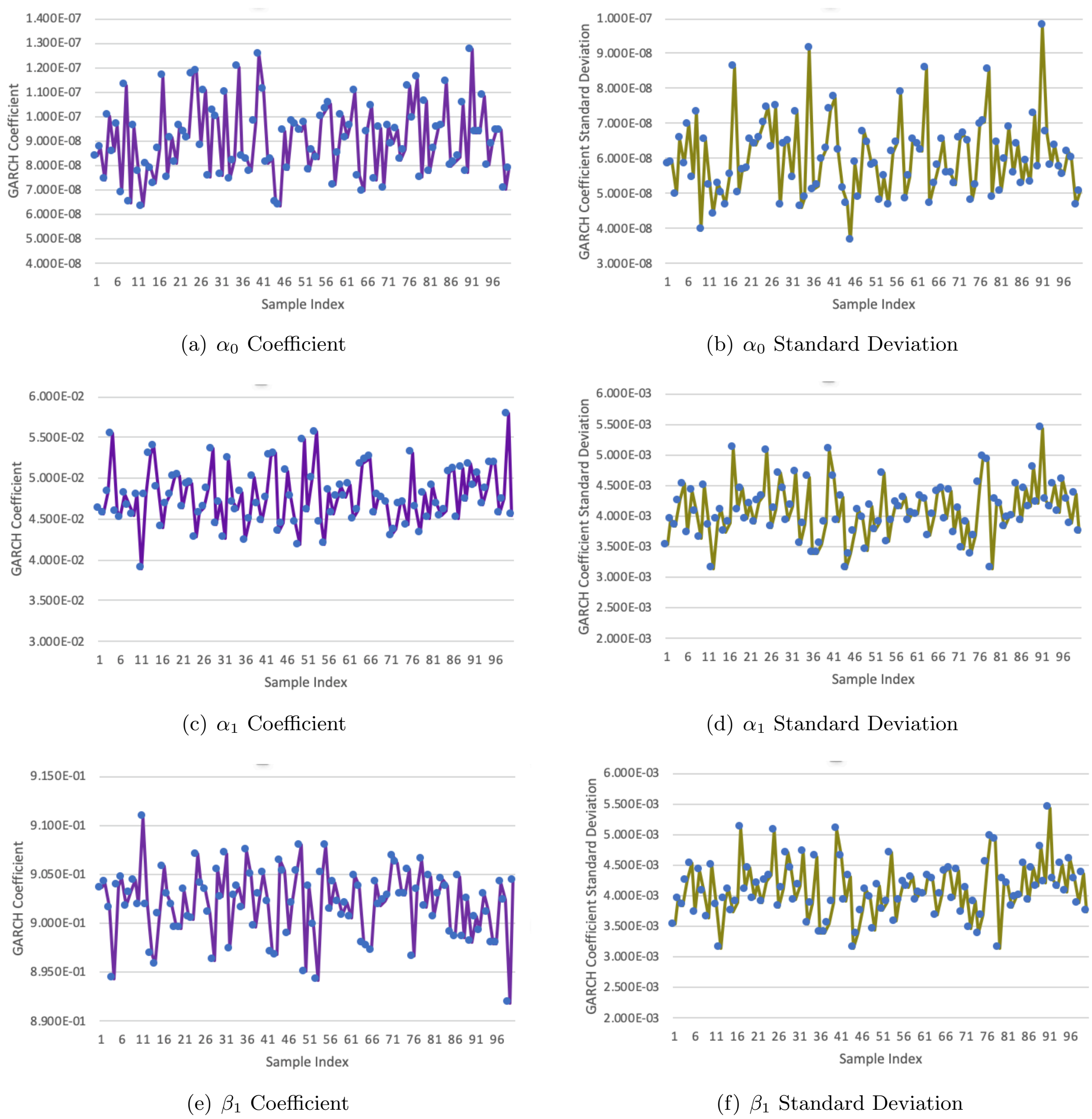


(a) ADF Test



(b) KPSS Test

Figure 3.27 ADF & KPSS of Estimated Squared Residuals of the Simulated Return Rate Series



(a) α_0 Coefficient

(b) α_0 Standard Deviation

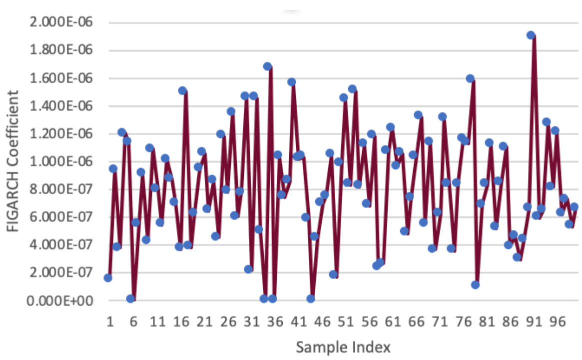
(c) α_1 Coefficient

(d) α_1 Standard Deviation

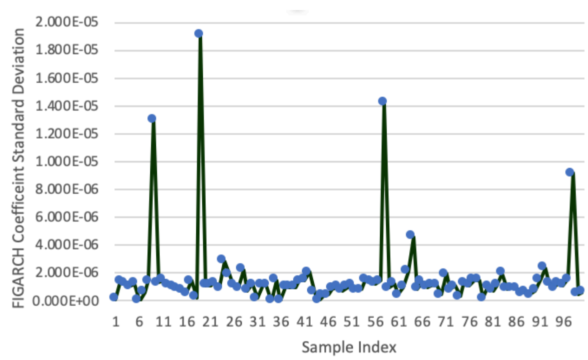
(e) β_1 Coefficient

(f) β_1 Standard Deviation

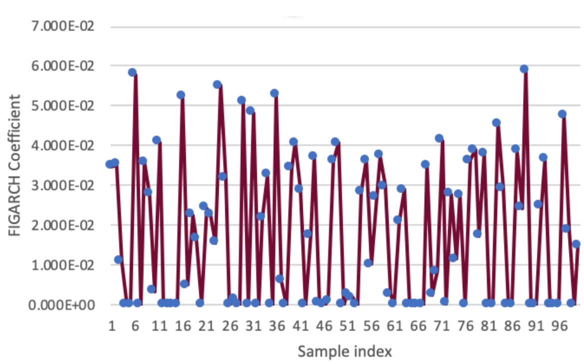
Figure 3.28 GARCH (1,1) Estimated Parameters of Simulated Return Rate Series



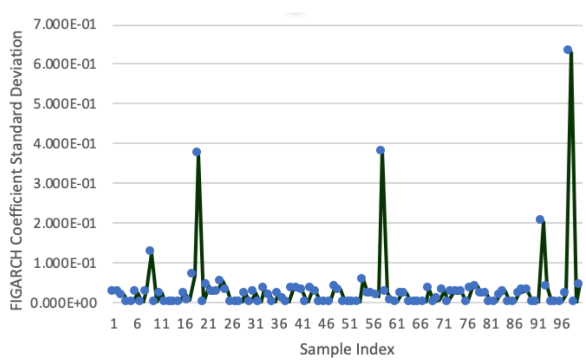
(a) α_0 Coefficient



(b) α_0 Standard Deviation

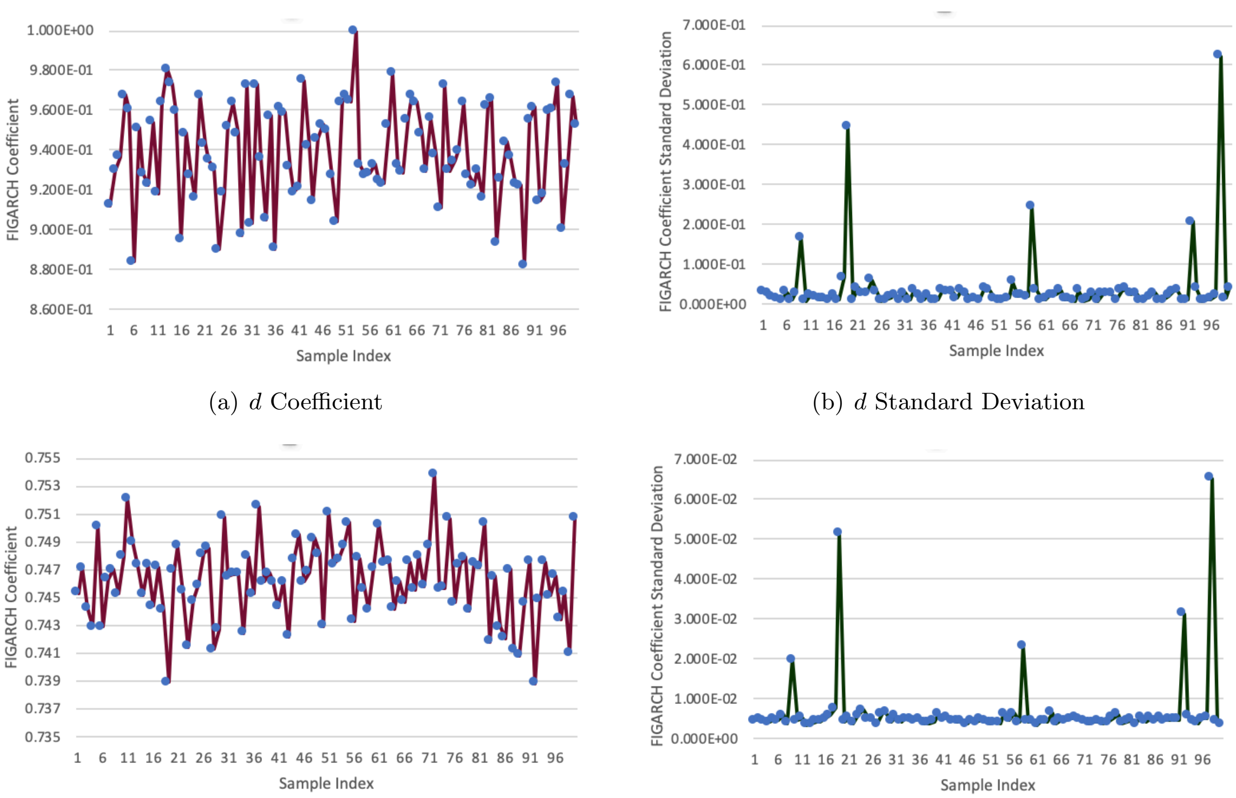


(c) ϕ_1 Coefficient



(d) ϕ_1 Standard Deviation

Figure 3.29 FIGARCH (1,1) Estimated Parameters of Simulated Return Rate Series A



(a) d Coefficient

(b) d Standard Deviation

(c) β_1 Coefficient

(d) β_1 Standard Deviation

Figure 3.30 FIGARCH (1,1) Estimated Parameters of Simulated Return Rate Series B

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CHAPTER 4. AN ECONOMIC MODEL OF BITCOIN MINING COMPETITION

To be submitted

Abstract

The motivation for this paper is to establish an economic model for Bitcoin mining behavior based on the Bitcoin protocol edited by [1]. In each period of Bitcoin mining competition, Bitcoin miners determine whether to participate in the mining competition or not and how much they should invest with reference to the Bitcoin spot market price, given the production information of aggregate miners. One-stage equilibrium of participating in Bitcoin mining competition is provided and an extension of multi-stage equilibrium of mining behaviour is described. The results show that for any Bitcoin miners, the equilibrium input only depends on the comparison of a miner's own marginal cost with that of the rest miners. However, whether the profit can be obtained or not depends on the miner‘s own fixed cost. This paper provides a benchmark for further research on mining competition in economics.

Keywords: Bitcoin Protocol, Bitcoin Mining Competition, Optimal Mining Strategy, Number of Miners

4.1 Introduction

Bitcoin system is the first peer-to-peer electronic payment system, of which the Bitcoin network, Bitcoin block chain and Bitcoin serving as an exchange medium are the three main components. Bitcoin system proposed by [1], works as a decentralized public ledger system to record the transactions. This decentralized structure prevents the individual agents from manipulating the transaction information in the public ledger, based on the fact that enough nodes participating in the mining competition and verifying the transaction information independently.

The nodes in the Bitcoin network are also called Bitcoin miners, who compete to solve the puzzle. After each round of competition, the miner who first solves the problem obtains the right to upload the block containing the transaction information to the block chain, and to get all the Bitcoin contained in the block with the related transaction fee. This process of obtaining Bitcoin is similar to traditional mining. The puzzles are solved with a specialized guessing. The professional chips designed for the Bitcoin mining competition perform the guessing calculation with a large consumption of electricity power. The more times a miner calculates within time unit, the more likely a miner solves the puzzle.

Following [2], this paper proposes a benchmark model of Bitcoin mining competition to analyze the miner's mining strategy, based on the Bitcoin protocol. The main contribution of this research is to propose an optimal strategy for the miners to determine the optimal level of investment, given the exogenous market price of Bitcoin. There exists an equilibrium level of resource input, which only depends on a miner's marginal cost compared to that of all the other miners, rather than the value of Bitcoin rewards that could be obtained. Moreover, even in an equilibrium state, the miners may have a negative profit due to the high fixed costs.

The remainder of this paper is organized as follows: Section 4.2 gives a brief description of Bitcoin system particularly on the characteristics of mining competition. Then a review of related literature is provided in Section 4.3. In Section 4.4, a theoretical model of Bitcoin mining competition is constructed. In Section 4.5, the theoretical model is simulated by the real data. Finally, the conclusion is made in Section 4.6.

4.2 A Description of Bitcoin System

4.2.1 Essential Component of Bitcoin System

Bitcoin network, **Bitcoin block chain** and **Bitcoins** are three dominant components of Bitcoin system. Bitcoin network is constructed as a "peer to peer" architecture based on internet. "Peer to peer" indicates that there is no hierarchy for the nodes which exist in Bitcoin network . All the nodes coordinate to maintain the service of Bitcoin network.

Block is a decentralized, distributed and public digital ledger that is used to record the transactions across the nodes in Bitcoin network. Each block links to its prior block and its subsequent blocks to assemble the block chain. "Decentralized" suggests that no authorized third-party institution verifies the transaction in this network. "Distributed" implies that the components, which communicate and coordinate their actions, are located globally at different computers in the network. "Public" implies the openness of transaction record in the block chain, although they are anonymous. In addition, Bitcoin is the reward to the node which wins the right to record transaction into the block chain. Moreover, a **Bitcoin protocol** is followed by all the nodes as a general rule which contains two major kinds of information: first, how to operate this decentralized system; second, how to maintain the security of the block chain which is a public ledger of transaction record.

4.2.2 Issue of Bitcoin

[1] sets the aggregate reservation of Bitcoin as 21 million which are enclosed in the blocks. Initially, there are 50 Bitcoins enclosed in each block as reward. This reward will reduce to half after every 210,000 blocks being created. Generating a new Bitcoin block requires approximately 10 minutes. Given this block time, it takes 1458.3 days or 3.99 years to generate 210,000 Bitcoin blocks and reduce the Bitcoin reward into half. Since total reservation of Bitcoin is 21 million, so 132 (210000 * 10/60/24/365) years are required to issue all the Bitcoins, which will be around AD 2140 from 2008 on-wards. Thereafter, the new generated blocks will no longer contain any Bitcoins.

The nature of transaction in Bitcoin system is data structure, which contains the information of the value transferred among Bitcoin users. In other words, a transaction refers to the information that a Bitcoin user authorises to transfer his or her Bitcoin to other users. Then through a new transaction, the receiver can transfer his or her Bitcoin acquired to others.

4.2.3 Transaction

Transaction size in Bitcoin system does not refer to the amount of Bitcoins involved in the transaction but the number of bits (characters) used to record this transaction. The input and output are two main components of Bitcoin transaction. Specifically, the input is a data structure referring to an output from a previous transaction which indicates the source of Bitcoin. The output is an instruction of sending Bitcoin, in which the information of the amount of Bitcoin and the public key to receivers is provided. Usually, the transaction fee paid to the Bitcoin miner is the difference between the amount of Bitcoin in input and that in output.

A typical transaction process in Bitcoin system can be described as follows. In order to purchase services or commodity by Bitcoin, a buyer first needs to check whether there exists enough Bitcoin in his or her Bitcoin address. This is the prerequisite of all transactions. For the purchaser, each payment is an output. The information on who receives this amount of Bitcoin is stored in the output. Since only the seller's private key can match this information, then the seller's Bitcoin address provides the unique signature to redeem the output and Bitcoin will only be transferred from the buyer to the seller. After the seller's Bitcoin address accepts this amount of Bitcoin, a corresponding transaction input will be created for him or her.

4.2.4 Bitcoin Mining Competition

The nodes in Bitcoin network which participate in the competition of block creation are defined as **Bitcoin miner**. The transactions, which are to be recorded, consist of the **memory pool** in Bitcoin network. During each 10-minute block-generating period, the individual miner collects and validates the transactions from memory pool and aggregates them into a candidate block. Simultaneously, these miners compete to solve **Hash puzzle**, in order to update their candidate block to be the unique valid one. When the miner selects the transactions from the memory pool and uploads them to form the candidate block, he or she maximizes the aggregate revenue of transaction fee under the restriction of one block's maximum capacity. However, the correlation between the transaction fee and the transaction size is not strictly positive linear. In other words, a transaction with a large transaction size might be attached with a low level transaction fee by the Bitcoin users. The optimal selection scheme for Bitcoin miner will be illustrated in Section 4.

According to the Bitcoin protocol, the winner, who solves Hash puzzle first, obtains the right to update his candidate block to be a valid block and uploads this valid block to the block chain immediately. Each round of competition is ended when all the other miners receive the announcement of win. These miners validate the result, which only requires one time of calculation to verify whether the answer is correct. If the answer is right, then all the miners propagate the result to the entire network independently. Therefore, this kind of work, namely, the mining is expensive, but the process of validation is very cheap. The winner of competition not only receives the transaction fees paid by the Bitcoin users but also receives Bitcoin enclosed in the block as reward.

Counts of **Hash guess**, which is used to solve the puzzle of block, determine the **difficulty** of competition. Bitcoin genesis block was created by Nakamoto's 232 times Hash guess and the number of 232 times is denoted as 1 difficulty. At each time period t, the level of difficulty can be defined as the amount of Hash guess for mining the block divided by that of mining the genesis block. Besides this, the Bitcoin protocol stipulates that the difficulty level of mining will be adjusted in each period of 2016 blocks added to the Bitcoin block chain, in order to maintain the time of one block generating as 10 minutes approximately. If the average time of mining for the previous added 2016 blocks is lower than this value, the difficulty level of the next 2016 blocks will increase. It implies that the level of difficulty in practice will be adjusted about every two weeks.

In [1], this 10-minute setting looks like a certain level of arbitrary, which made the transfer faster than that of a common bank-to-bank transfer at the time. Besides, this 10-minute period allows the winning miner has enough time to upload the candidate block to the block chain and join in the next round of competition.

The mining competition is of great significance to Bitcoin system. With the increasing number of blocks mined out, tampering with the information recorded in the block chain becomes much more difficult. Specifically, if a cheater wants to change the records of a past transaction, his or her Hash guessing power not only has to exceed that of all other miners who maintain the growth of the legitimate block chain, but also must be large enough to recalculate all blocks after the manipulated block. This requires a large amount of resource (electricity power). Since the Bitcoin protocol only admits the longest chain in the network and only accepts the transaction information on that chain, then it becomes increasingly difficult to modify the transaction information as the block chain extends.

Since the security of Bitcoin block chain depends on the amount of computational time spent in mining competition of Bitcoin system, a positive feedback effect appears. The more users joining in Bitcoin system, the higher the value of Bitcoin would be, the more Bitcoin miners will be attracted due to the increasing rewards. Eventually the security of block chain will be enhanced which will attract more users to use Bitcoin. It is worth noting that security is one of the factors that determine the value of Bitcoin system while the price of Bitcoin is determined in the trading platform, which is exogenous to mining competition.

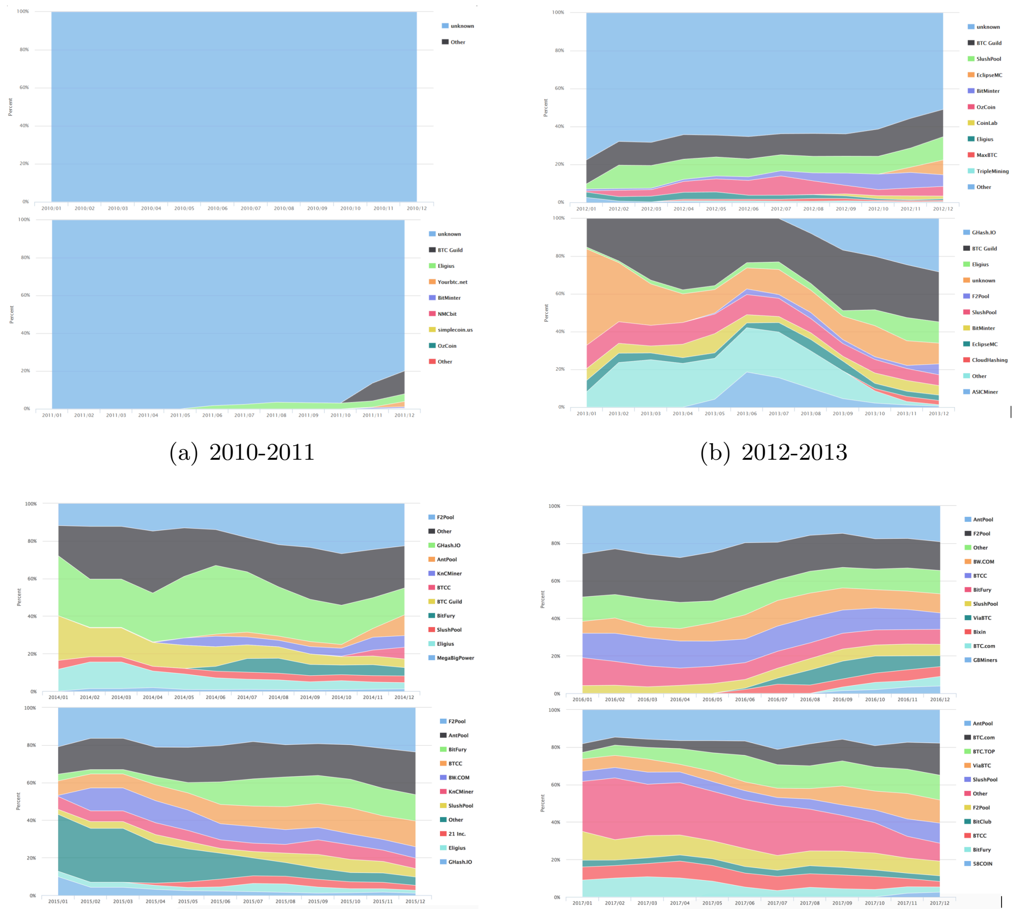
4.2.5 Mining Pool

In practice, Bitcoin mining competition is so competitive that the individual miner has no chance to win independently because the individual miner can hardly compete with large scaled groups of miners with tens of thousands of chips and hydro-power stations providing the cheap electricity power. The miners have limited time to mine before the equipment is replaced by the next generation mining machines with more Hash power. As a result, the individual miners cooperate to form a **mining pool**, which aggregates all the participants' computing power together to mine and share rewards.

By participating in the mining pool, each miner receives a small partition of the overall return. Although this reward may no be very profitable, but daily profits are guaranteed, which reduces the uncertainty for the individual miners. Besides, this regular income helps the individual miner to amortize the fixed cost (investment in equipment). Thus, even if the probability that equipment becomes outdated after one or so is high, the income is stable and reliable, at least during this period.

The historical evolution of mining pool is showed in following Figures 4.1. In plot (a) of the figure, all the miners worked individually before 2011. The first mining pool appeared in May 2011. However, at that period, the maximum aggregate Hash power of all mining pools only accounted for 20% of the whole net computing power (December 2011).

The proportion of Hash power of mining pools has increased, especially for the top 10 largest mining pools. From 2014 on, the aggregate Hash power of the small mining pools and the individual miners has seldom exceeded 25%. Hence, this market structure tended to be a monopolistic competition rather than a perfect competition. Moreover, it is noted that the top ten largest mining pools have been changing. The reason is that mining pool is a very loosing organization and is only connected by servers and internet. Therefore, the miner might select different mining pools to participate in different years. This situation causes that the locations of mining pool registration and do not match with the miner's specific equipment, and the corresponding electricity costs of two locations may be completely different.



(a) 2010-2011 (b) 2012-2013

(c) 2014-2015 (d) 2016-2017

Figure 4.1 Hash Distribution of Bitcoin Mining Pool 2010-2017

4.2.6 Summary

The key properties of Bitcoin system is summarized as follows. The transactions in Bitcoin system refer to using Bitcoin as an exchange medium to transfer value or wealth among users. The essence of Bitcoin transaction is the information of the value transfer, which will be recorded in the decentralized, distributed ledger (*i.e.* Bitcoin block chain). All the nodes in Bitcoin network maintain the same distributed ledger that deals with transactions synchronously. No authoritative node has ability to control the system. During each 10 minute interval, a new node which is the winner of mining competition obtains the right to process and record transactions. The balance transferred through Bitcoin system can only be changed through the valid transactions which verified by all the other miners and the history of transactions cannot be removed from the block chain.

4.3 Literature Review

This paper contributes to the economic literature on cryptic-currency and related block-chain technology, specifically on the development of a theoretical model for Bitcoin mining competition.

4.3.1 Anonymity & Security

[1] introduces the anonymization mechanism for transactions in the Bitcoin network. A series of literature ([3], [4], [5] and [6]) study the efficiency of anonymization. Most of research propose that anonymization mechanism of Bitcoin network is partially effective. There exists the possibility of privacy exposure and de-anonymization using Bitcoin to purchase goods or services. In addition, the organization of mining pool increases the risk of error message transmission in the Bitcoin network. Moreover, some other literature ([5], [7], [8]) propose the potential program of attacking the anonymity of Bitcoin system. For example, the Bitcoin users' public keys can be linked with their IP addresses through the progress of transaction propagation in Bitcoin network, with an accuracy of 30% approximately.

A series of literature proposes new technologies and alternative mechanism aimed at enhancing the privacy and anonymity of the Bitcoin system without disrupting its protocol which can be broadly classified into three categories: peer-to-peer mixing protocols ([9]; [10]; [11]); distributed mixing networks, ([12] and [13]) and Bitcoin extensions or Altcoins ([14] and [15]).

Besides, some other literature study the security of Bitcoin system. [16] and [17] investigate whether Bitcoin protocol can sustain the Bitcoin system against the external network attacks. They conclude that avoiding global 51% Hash power aggregating may not guarantee that the block chain will not be tampered. Bitcoin system is reliable only if the miner is too small to have any market power.

4.3.2 Mining Behaviour

4.3.2.1 Mining Process

The Bitcoin mining competition leads to the extensive discussions in the field of economics. A simple but intuitive model which illustrates how to mine in a Bitcoin system is constructed in [2]. At each time point, participating in the mining competition requires that the mining cost should not be greater than the expected reward of mining competition, namely, the probability of winning times the value of mined out Bitcoin. The probability of win depends on the aggregate amount of Hash power provided by all the miners at each time point. Moreover, the Bitcoin miners need to amortize the sunk cost brought by the investment in equipment. As a result, the miners may over-invest to offset their fixed costs until the investment equals to the marginal cost per time unit.

The possible "death spiral" in Bitcoin system is pointed out in [2] as well. As known, the equipment applied to Bitcoin mining is purchased by the fiat currency, and the price of Bitcoin is determined in the trading platform. Then the reward of Bitcoin mining measured in fiat currency fluctuates with the market price of Bitcoin. If the Bitcoin price falls dramatically due to the loss of confidence in Bitcoin system, then the incentives for the Bitcoin mining will also decline. This leads to the possibility of a "death spiral": a drop in Bitcoin price reduces the mining incentive and participation rate of Bitcoin mining, and the lower mining participation rate will make the protocol of Bitcoin system more vulnerable to be attacked, which results in further loss of confidence in Bitcoin system.

4.3.2.2 Mining Pool

The research on the Bitcoin mining pool mainly focuses on the task and reward allocation mechanism through game theory. The mining pool is a loosen organization in which the individual miners seek to increase the probability of winning in the competition and collaborate to mine. However, some literature ([18], [19], [20]) find that the goal of each individual miner is often inconsistent with that of the mining pool. Therefore, some miners have the incentive to switch among pools in order to maximize their benefits. For the mining pool, it is difficult or even impossible to distribute mining rewards in fixed or regular ways. Moreover, sometimes the mutual attacks occur among the mine pools, during the Bitcoin mining competition. A series of literature ([21], [22], and [23], [24]) research on the attacking events among the pools. They claim that the organization like mining pool violates the intention of [1] which is to design a decentralized Bitcoin operation system. The Hash power of the Bitcoin network has been concentrated after the individual miners joined in mining pool, which becomes a threat to the Bitcoin protocol.

4.3.3 Transaction Fee

Some other studies investigate the impact of Bitcoin users' transaction fee on the mining competition. Due to the limited capacity of each block, the transaction fee provided by the Bitcoin users is influenced by how quick the users desire the transaction to be verified; the size of the transaction and the amount of other transactions waiting in the memory pool. Therefore, how to construct an optimal candidate block is an interesting issue. [25] is the first economic literature that analyzes the optimal strategy in Bitcoin mining competition, but he mistakenly thought that miner's aggregating transaction to form a candidate block would reduce the speed to solve the Hash puzzle. Neither the Bitcoin users nor the transaction fee are included in the model of this research. [26] proposes that restricting the block size may lead to a self-adjusting market for the transaction fee, in which the Bitcoin miners can perform the optimal strategy of selecting the transactions. Besides, the queuing theory, which was proposed by [27], is introduced to analyze the Bitcoin mining behaviour ([28], [29] and [30]). In [28] and [29], the transaction fee attached by the Bitcoin users depends on their loss of waiting. Additionally, [30] focuses on how the small transaction fee affects the time required for the transaction confirmation. In this study, a model for Bitcoin transaction confirmation process, which works as a priority queuing system, is built, and a corresponding average transaction-confirmation time is proposed as well.

4.3.4 Summary

The literature cited are mostly focused on the anonymity effectiveness and security of Bitcoin system, the economic analysis of Bitcoin mining behaviour and the impact from mining pool organization, and how to determine transaction fee in Bitcoin system. Among the literature, [2] is the most related cited and the model illustrated in following Section 4 can be viewed as the extension and refinement of the model by [2].

4.4 Theoretical Model of Mining

Although the miners's motivation of participating in mining competition is to obtain Bitcoins and the transaction fees, for Bitcoin system, the essence of mining competition lies on the consolidation of the decentralized clearing transaction mechanism rather than creating new coins (*i.e.* the exchange medium). The Bitcoin mining process maintains the security of block chain and fulfills the consensus of the whole network without a centralized authority. The issuance of Bitcoin and the transaction fee attached supply the exchange medium to Bitcoin system, and align the miners' action with protecting Bitcoin system simultaneously.

In this section, a theoretical model of Bitcoin mining competition following [2]'s work is developed. The mining decision problem can be formulated in the following way. The Bitcoin miners observe the Bitcoin market price. To participate in mining competition, the miners have to invest their capital in professional mining equipment. The objective of the Bitcoin miners is to maximize their expected profits in a perfect competitive market.

4.4.1 Expected Profit

According to the Bitcoin protocol, the mining difficulty is maintained as approximately 10 minutes of generating a new block. With focus on the main characteristics of the mining competition, the minor time difference required to generate each block is ignored. During each period t, one block is mined out. In addition, Q_t, which is the number of Bitcoins contained in the block, is an exogenous variable. Moreover, in this model, the Bitcoin mining industry is competitive, in which each miner works independently and has no market power.

Bitcoins, which are included in the blocks, are encrypted. Bitcoin miners compete to solve the puzzle by the Hash guess. They need to complete this process with the professional mining hardware which consumes a lot of resource. Consider the Bitcoin miner i, his or her capital input is denoted as $k_{i,t}$ which provides $H_{i,t}$ Hash guess at time t:

$$H_{i,t} = h_{i,t}(k_{i,t}) \tag{4.1}$$

The Bitcoin miner i competes with other miners using this amount of Hash guess, in order to win the block. Since the performance parameters of professional mining equipment are easy to obtain, the form of function $h_{i,t}$ is public information to all the miners.

To simplify the model, the electricity is assumed as the only resource of input. The Bitcoin miner i's profit expectation is written as:

$$\pi_{i,t} = \Pr_{i,t}\{\text{Win}\}\left(p_t\left(Q_t + \Phi_{i,t}\right) - C_{i,t}\right) + \left(1 - \Pr_{i,t}\{\text{Win}\}\right)\left(-C_{i,t}\right) \tag{4.2}$$

Here, $\Pr_{i,t}\{\text{Win}\}$ is the miner i's probability of win. $\Phi_{i,t}$ is the exogenous aggregate transaction fees. The determination of $\Phi_{i,t}$ is illustrated in later sub-section 4.5. p_t is the exogenous spot market price of Bitcoin and $C_{i,t}$ is his or her mining cost in period t.

4.4.2 A Static Equilibrium

Since all the miners in Bitcoin system need to compete through the specific SHA265 Hash function, then the amount of time spent on solving Hash puzzle for all the miners follows the same distribution in each 10-minute block generating period. Therefore, the only way to solve the puzzle faster than the other miners is to increase the number of Hash guess per unit time by increasing Hash power. For the Bitcoin miner i, his or her probability of win at period t is determined by the proportion of his or her individual Hash power to the aggregate Hash power provided by the total M_t miners in this period:

$$\Pr_{i,t}\{\text{Win}\} = \frac{H_{i,t}}{\sum_{j=1}^{M_t} H_{j,t}} \tag{4.3}$$

Without loss of generality, the form of Hash function $H_{i,t}(k_{i,t})$ is formulated as:

$$H_{i,t} = h_{i,t}(k_{i,t}) = \theta_t k_{i,t} \tag{4.4}$$

where θ_t is an exogenous constant parameter which measures the marginal Hash guess provided by the miner's electricity power input.

The development of marginal Hash guess ability is an interesting issue in the field of Bitcoin mining. The Hash power provided by the mining hardware has grown exponentially from the birth of Bitcoin. In the early stage, everyone could simply use their personal laptops to mine Bitcoin and the Hash power provided by these laptops was low. Then between 2010 and 2011, many miners began to upgrade their equipments from CPUs to GPUs and then to "FGPA" (Field Programmable

Gate Array) to perform mining. Finally, the professional Bitcoin mining (*i.e.* ASIC mining) machine was invented in 2013. With introduction of ASIC mining, SHA256 algorithm for mining was directly solidified on the dedicated silicon chip which led to another huge leap in the growth of Hash power. At this moment, one professional mining hardware can provide greater Hash power than that of the whole network of 2010. Therefore, θ_t in equation (4.4) evolves with time.

Moreover, all the Bitcoin miners are assumed to use the latest mining hardware, namely, θ_t is the same for all the miners. Since the mining competition is very competitive, it is impossible to win the competition with the outdated equipment. Therefore, the most effective way for the miners to increase their probability of win is to increase Hash power through purchasing more advanced equipment and inputting more electricity power. Besides this, the situation, in which the manufacturers of designing the advanced mining hardware are not considered as participants. Therefore, the mining technology can not be monopolized.

In practice, the persistent enhancement of Hash guess ability in Bitcoin mining hardware has raised the barrier of participating in Bitcoin mining competition. The miners not only need to find the cheapest electricity power but also have to spend a large amount of money to purchase or update the mining machines (fixed cost). Therefore, the cost function of miner i is formulated as:

$$C_{i,t} = c_{i,t}k_{i,t} + \bar{F}_{i,t} \tag{4.5}$$

where $c_{i,t}$ is the marginal cost which is various across the miners. $c_{i,t}$ can be viewed as the price of electricity power of miner i in period t. $\bar{F}_{i,t}$ represents the fixed cost of mining game which evolves with time t including the expenditure on purchasing machine and site rental.

Given the specific form of cost function, the miner i's profit expectation is obtained by combining equations (4.2) - (4.5):

$$\pi_{i,t} = \frac{k_{i,t}}{\sum_{j=1}^{M_t} k_{j,t}} p_t(Q_t + \Phi_{i,t}) - c_{i,t}k_{i,t} - \bar{F}_{i,t} \tag{4.6}$$

In equation (4.6), the parameter θ_t is eliminated, indicating that the marginal Hash guess provided by the mining machine does not affect the miner's profit expectation in a highly competitive Bitcoin mining industry at period t.

Take the first order condition of (4.6) with respect to $k_{i,t}$, then:

$$c_{i,t} = \frac{p_t\left(Q_t + \Phi_{i,t}\right) \sum_{j=1,j\neq i}^{M_t} k_{j,t}}{\left(\sum_{j=1}^{M_t} k_{j,t}\right)^2} \tag{4.7}$$

Sum up the marginal cost $c_{i,t}$ of all M_t ($i \in 1, ..., M$) miners and rearrange, then we have:

$$\sum_{j=1}^{M_t} k_{j,t} = \frac{p_t\left(Q_t + \Phi_{i,t}\right)\left(M_t - 1\right)}{\sum_{j=1}^{M_t} c_{i,t}} \tag{4.8}$$

Through equations (4.7) and (4.8), the equilibrium amount of the electricity power input by the miner i is solved out:

$$k_{i,t}^{*} = \frac{p_t\left(Q_t + \Phi_{i,t}\right)\left(M_t - 1\right)\left[\sum_{j=1}^{M_t} c_{j,t} - c_{i,t}\left(M_t - 1\right)\right]}{\left(\sum_{j=1}^{M_t} c_{j,t}\right)^2} \tag{4.9}$$

This is the first equilibrium equation for the Bitcoin mining behaviour. The number of Bitcoins contained in block Q_t is exogenous and the transaction fee $\Phi_{i,t}$ is determined independently. This equation indicates that the required capital of mining competing for miner i at period t depends on the total number of miners participating in the competition M_t, their electricity prices (the marginal costs) $c_{j,t}$, $(j \in 1,...,M_t)$ and market price of Bitcoin p_t. Besides, if the Bitcoin miner i has decided to participate in the mining competition, then $k^*_{i,t}$ should be positive. Therefore, it is required that $\sum_{j=1}^{M_t} c_{j,t} - c_{i,t}(M_t - 1) > 0$. This condition indicates that participating in mining decision depends on its marginal cost compared with other miners, but not the potential mining reward.

By substituting $k^*_{i,t}$ with (4.9) in (4.6), the expected profit of Bitcoin miner i is:

$$\pi^*_{i,t} = \left(\frac{\sum_{j=1}^{M_t} c_{j,t} - (M_t - 1)c_{i,t}}{\sum_{j=1}^{M_t} c_{j,t}} \right)^2 p_t \left(Q_t + \Phi_{i,t}\right) - \overline{F}_{i,t} \tag{4.10}$$

Equation (4.10) indicates that given the market price of Bitcoin and the fixed cost, the profit of miner i depends on his or her unit cost of electricity, the number of other miners and the cost of electricity. When $\overline{F}_{i,t} < \left(\frac{\sum_{j=1}^{M_t} c_{j,t} - (M_t - 1)c_{i,t}}{\sum_{j=1}^{M_t} c_{j,t}} \right)^2 p_t \left(Q_t + \Phi_{i,t}\right)$, the Bitcoin miner i has a positive profit. Otherwise, the miner would suffer from a loss.

The competition at equilibrium can be illustrated by a specific numerical example. Suppose at time t, three Bitcoin miners participate in the competition, *i.e.*, M_t equals to 3. W.L.O.G, their marginal costs $c_{1,t}$, $c_{2,t}$ and $c_{3,t}$ are set as 2μ, 3μ and 4μ respectively. According to equation (4.9), the equilibrium inputs for these three miners are: $k^*_{1,t}$ equals to $\frac{10p_t(Q_t+\Phi_{1,t})}{81\mu}$, $k^*_{2,t}$ equals to $\frac{2p_t(Q_t+\Phi_{2,t})}{27\mu}$ and $k^*_{3,t}$ equals to $\frac{2p_t(Q_t+\Phi_{3,t})}{81\mu}$. Then through equation (4.10), the corresponding profit of these three miners $\pi^*_{1,t}$, $\pi^*_{2,t}$ and $\pi^*_{3,t}$ are $\frac{25p_t(Q_t+\Phi_{1,t})}{81} - \bar{F}_{1,t}$, $\frac{16p_t(Q_t+\Phi_{2,t})}{81} - \bar{F}_{2,t}$ and $\frac{p_t(Q_t+\Phi_{3,t})}{81} - \bar{F}_{3,t}$ respectively. Therefore, these three level of profit can not be compared without the information of the fixed costs. If the fixed cost of the first miner is higher with the lowest marginal cost, his or her profit will probably be lower than the other two miners.

If in period $t + 1$, the third Bitcoin miner, who has the highest marginal cost in previous period, decreases its marginal cost to 0.5μ, while the other two miners keep their marginal costs the same. Then the second miner will not participate in the competition due to $\sum_{j=1}^{M_{t+1}} c_{j,t+1} - c_{i,t+1}(M_{t+1}-1) = 5.5\mu - 6\mu < 0$. In this situation, only the first and third Bitcoin miner would participate in the mining competition. Their equilibrium inputs $k^{*'}_{1,t+1}$ and $k^{*'}_{3,t+1}$ are $\frac{2p_{t+1}(Q_{t+1}+\Phi_{1,t+1})}{25\mu}$ and $\frac{8p_{t+1}(Q_{t+1}+\Phi_{3,t+1})}{25\mu}$. The corresponding profits are $\frac{p_{t+1}(Q_{t+1}+\Phi_{1,t+1})}{25} - \bar{F}_{1,t+1}$ and $\frac{16p_{t+1}(Q_{t+1}+\Phi_{3,t+1})}{25} - \bar{F}_{3,t+1}$, respectively. Therefore, for the third miner, there exists a strong incentive to reduce marginal cost if the expenditure on reducing the cost is smaller than the potential incremental profit, given the mining reward and fixed cost stay the same.

This example also shows that no matter how the Bitcoin miners reduce their marginal costs, as long as the initial total number of miners in mining competition was greater than two, then at least two miners will participate in the mining competition at each period, although in some cases, the Hash power concentration may be greater than 50%.

4.4.3 Two Extreme Cases

Consider an extreme case that there exists only one miner in the system for some reason. Bitcoin block chain will become a centralized block chain. In equations (4.9) and (4.10), the electricity inputted approaches to zero and the net profit is $p_t(Q_t + \Phi_{i,t}) - \overline{F}_{i,t}$. If $p_t(Q_t + \Phi_{i,t}) > \overline{F}_{i,t}$, then this positive profit will attract other nodes to become miners to participate in the mining game. Therefore, in this case, the security of Bitcoin system is influenced by the market price of Bitcoin implicitly.

Moreover, consider another specific case that all the Bitcoin miners are identical and the mining game is standard free entry-exit. This case helps to estimate the number of miners who participate in the mining game. "Identical" indicates that all the miners use the same technology and access the same price of electricity power.

Therefore, each miner's expected profit converge to zero when no potential miner enters or exits the game. Then, equation (4.10) is changed to:

$$\pi_t^* = \left(\frac{1}{M_t}\right)^2 p_t\left(Q_t + \Phi_t\right) - \overline{F_t} = 0 \tag{4.11}$$

The equilibrium number of miners in this situation is:

$$M_t^* = \left\lceil \left(\frac{p_t\left(Q_t + \Phi_t\right)}{\overline{F_t}}\right)^{\frac{1}{2}} \right\rceil \tag{4.12}$$

where $\lceil\rceil$ is a ceiling function. When $p_t\left(Q_t + \Phi_t\right) > \overline{F_t}$, there are at least two miners in the system that sustains the block chain.

In this case, the probability of win for each miner can be solved out as well:

$$\Pr_t\{Win\} = \frac{1}{\left\lceil \left(\frac{p_t(Q_t+\Phi_t)}{\overline{F_t}}\right) \right\rceil} \tag{4.13}$$

Till this point, the model of Bitcoin mining competition is completed. It is notable that the idea of this model is based on the principle of Bitcoin system described in [1]. The equilibrium level of capital input and the profit for Bitcoin miner are formulated explicitly. However, to some aspects, the realized Bitcoin mining competition is different from those described in our theoretical model. For example, the mining pool organization has certain degree of market power. Therefore, this model can be viewed as a benchmark model that can be extended to analyze the realistic mining problems.

4.4.4 A Dynamic Equilibrium

Following the Bitcoin protocol, a static equilibrium of Bitcoin mining competition at time point t is developed. However, a class of Bitcoin nodes specialize in the mining competition. Since this class of miners are concerned on the whole life profit, then they tend to stay in the mining competition even when suffering the lost (*i.e.* the negative profit) in certain periods, unless their total resources are exhausted. Given an initial endowment of this type of Bitcoin miners as I, he or she chooses an optimal input strategy to maximize:

$$\Sigma_{t=1}^{\infty}\beta^t\Pi_{i,t} \tag{4.14}$$

where

$$\pi_{i,t} = \frac{k_{i,t}}{\sum_{j=1}^{M_t} k_{j,t}} p_t(Q_t + \Phi_{i,t}) - c_{i,t}k_{i,t} - \bar{F}_{i,t} \tag{4.15}$$

s.t.

$$c_{i,t}k_{i,t} \leq I_i - \Sigma_{\tau=1}^{t-1} c_{i,t}k_{i,t} + \Sigma_{\tau=1}^{t-1}\Pi_{i,t} \tag{4.16}$$

Given the exogenous price sequence $\{P_t\}_{t=1}^{\infty}$, the number of Bitcoins contained in block $\{Q_t\}_{t=1}^{\infty}$ and the marginal cost of other miners c_j, a strategy $\left\{k_{i,t}^*\right\}_{t=1}^{\infty}$ defined in equation (4.9) with the constraint equation (4.16) consists of a dynamic equilibrium.

4.4.5 Determination of the Optimal Transaction Fee

In this sub-section, how the Bitcoin miners optimize the potential aggregate transaction fees is analyzed. During each period of generating one block, the Bitcoin miners select the transactions from the memory pool to generate their own candidate blocks. The miner maximizes the aggregate transaction fees subject to the capacity constraint of each block. For each transaction, the two key attributes are the transaction size (bytes) $\psi_{j,t}$ and the attached fee $\Phi_{j,t}$, both of which are denominated in Bitcoin.

The attached fee is not strictly positive correlated with the transaction size since some Bitcoin users do not require their transactions to be verified immediately. Some transaction fees can be zero if delaying the verification does not have an effect. Denote Θ as the maximum capacity of each block and ε_{ψ} as the lower bound of size of any individual transaction. Then the solution algorithm for this problem consists of the following steps:

Step 1. For each transaction j, the Bitcoin miner computes the ratio $\alpha_{j,t-1}=\frac{\psi_{j,t-1}}{\Phi_{j,t-1}}$;

Step 2. The Bitcoin miner sorts $\alpha_{j,t-1}$ in a decreasing order, and indexes them by $j\in \{1,...J\}$;

Step 3. The Bitcoin miner starts from the first transaction $j = 1$. If $\psi_{1,t} > \Theta$, then the miner drops the 1st transaction and proceeds to the 2nd transaction; if $\psi_{1,t} < \Theta - \varepsilon_{\psi}$, then the miner selects this transaction and proceeds to the next transaction;

Step 4. $\bar{J}$ is denoted as the set of transactions that have been selected up. The miner determines the termination criterion in this way: Check whether $\sum_{j\in\bar{J}} \psi_{j,t}+\psi_{n,t}> \Theta$. If true, then proceed to $n + 1$; if otherwise, then select transaction with index n and stop.

In this way, Bitcoin miner constructs the optimal candidate blocks, given the exogenous transaction size and attached transaction fees.

4.5 Simulation

In this section, the calibration to Bitcoin mining model constructed in Section 4 is performed. The realized data are applied to simulate the equilibrium equations. However, there are two reasons that make it difficult to calibrate these two equilibrium equations (4.9) and (4.11) directly.

First, the Bitcoin miners are located globally around the world, then the marginal cost of each miner $c_{i,t}$, namely, the price of electricity power is difficult to obtain and compare. Second, the miners collaborate and form the mining pool to compete, such that the number of miners M_t can not be observed directly.

All these difficulties make it impossible to simulate through equations (4.9) and (4.11). Therefore, the simulation through equation (4.12) is implemented. This equation determines the equilibrium number of miners given an extreme situation that all Bitcoin miners are homogeneous and the market is free entry and exit.

The data source for the simulation is obtained from the white paper provided by company "Canaan" which would like to seek "Initial Public Offerings" (IPO) but failed in Hong Kong stock exchange market. "Canaan" is a computer hardware manufacturer specialized in the design of integrated circuits for Bitcoin mining, and is headquartered in Beijing, China. The product type of "Canaan" is named as "Avalon". Figure 4.2 shows the homepage of this company.

Avalon A3256 chip was developed in January 2013, which was the first chip specifically designed for Bitcoin mining competition, representing the beginning of an era of specialized Bitcoin mining hardware. In the early stage of the specialized Bitcoin mining, a lot of companies participated in the design and sale of the specialized mining hardware. Designing and selling the professional mining hardware is very profitable, due to the fact that Bitcoin miners are very eager to pay premiums for the most advanced equipment. However, if the period of "Research and Development" (R&D) is so long that the new developed equipment might not catch up with the most advanced Hash computing power, moreover, the R&D requires tremendous investment. Therefore, most of the companies quit from this industry because of the extremely intense competition.

Figure 4.2 Product of Canaan (Avalon)

Currently, the three largest mining chip manufacturers monopolize the supply of more than 90% Hash guess power of the whole network. They are BITMAIN (Product type: “AntMiner” series), Canaan (Product type: “Avalon” series) and Ebang (Product type: “Ebit” series). In other words, for a potential Bitcoin miner, he or she is very likely choose chips from these three companies to participate in mining competitions.

Among these three companies, the vendor “Canaan” sought IPO but failed in 2018. The information about the production and sales from 2013 to 2017 is listed in its white paper prepared for IPO. “Canaan” concentrated themselves on the design and sale of the mining-specific chips in the early development. The types of chips include: A3256, A3255, A3233, A3222. Then "Canaan" switched to provide the integrated mining equipment. The types of the equipments are: A6, A721, A741, in which the chips are the core part.

The sales information of Avalon series products including chips and integrated equipments is summarized in Table 4.1. The sale period of each product, the corresponding retail price and the total amount of sales are provided.

Table 4.1 Selling Information of Avalon Series

Period	2015	2015	2016	2016	2017 Jan-Apr	2017 Jan-Apr
Product	Series of Chips	A6	A6	A721	A721	A741
Selling Amount	804,560	9,727	72,836	20,918	9,578	48,233
Retail Price (Renminbi)	24	3,673	3,159	4,098	3,739	4,512

On the other hand, since specific parameters of all Avalon series product, such as chip thickness and the amount of Hash guessing per second, are public information, then we can calculate the sum of Hash power provided by the Avalon series product.

Besides this, the proportion of Hash power from Avalon series product can also be calculated since the aggregate Hash power of whole Bitcoin network has been recorded as public information as well. Figure 4.3 shows the history of Bitcoin network from January 2^{nd}, 2015 to April 29^{th}, 2017. Here, the unit of Hash power is Th/s. As can be seen from the figure, Hash power of the whole network has been increasing rapidly.

In following Table 4.2, the ratios of each Avalon product's Hash power to Bitcoin total network Hash power at each period are listed respectively.

Let us recall the components of equation (4.12). The market price of Bitcoin can be observed directly. Amount of Bitcoin and transaction fees in each period can be found from major block

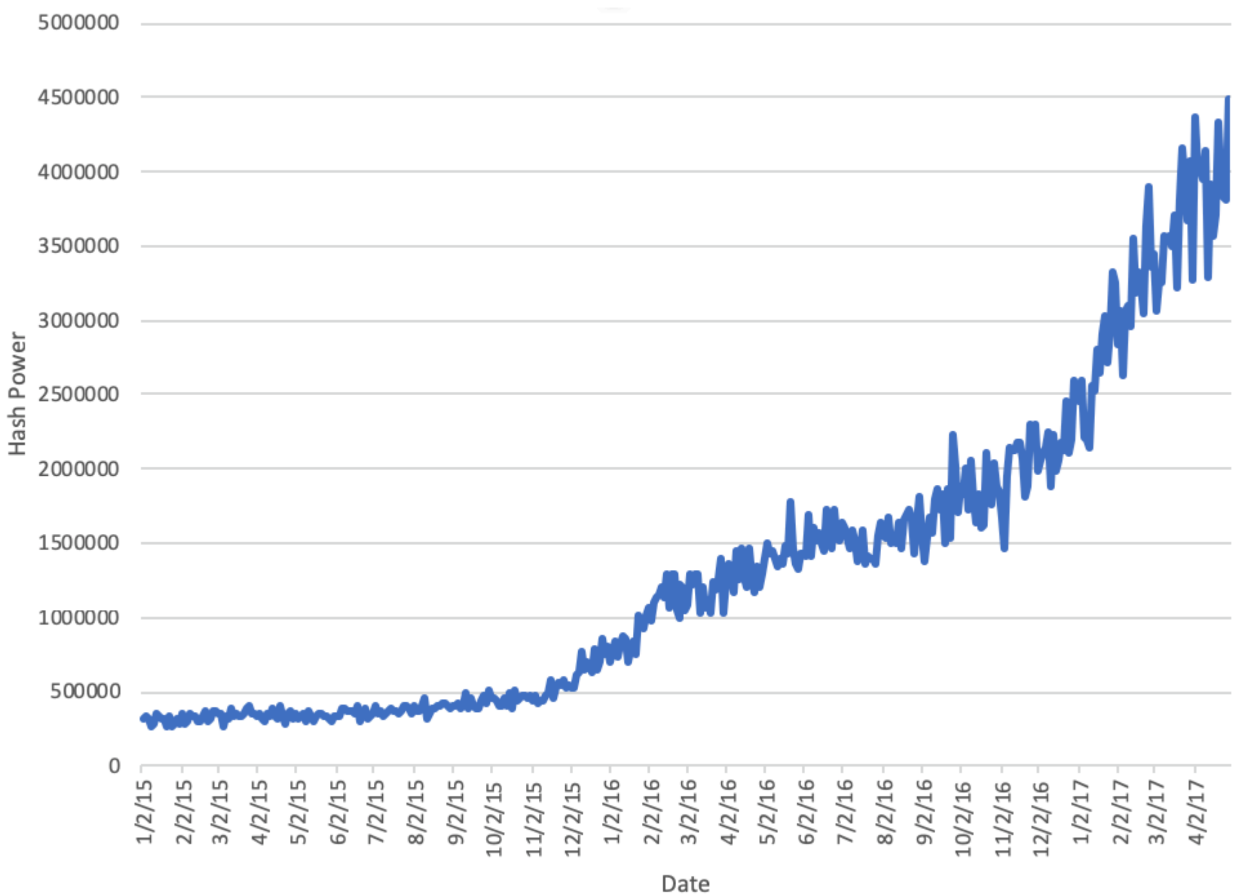


Figure 4.3 Historical Hash Power of Whole Bitcoin Network

chain websites. Besides this, the purchase of Bitcoin mining equipment is considered as the only source of fixed cost of mining input. Therefore, the number of Avalon mining equipment that used in mining competition can be simulated through equation (4.12).

There are three important assumptions implied in this scheme of simulation. First, we assume that the number of mining equipment equals to that of Bitcoin miners. It does not have much impact on the conclusions, because our simulation focuses on the agency that provides Hash power. Second, other components of fixed costs are not included, such as the site rental fees. However, the cost of land leases is not a major source of cost in most cases of Bitcoin mining, since large-scale

Table 4.2 Parameters of Avalon Product

Type	Issue Date	Hash Power	Die	Selling Amount	Aggregate Proportion	Individual Proportion
From 2014/12/31-2015/12/31						
Avalon A3256 Chip	Jan-2013	295	110nm	40,228	-	$2.420e^{-05}$
Avalon A3255 Chip	Aug-2013	1,500	55nm	40,228	-	$1.240e^{-04}$
Avalon A3233 Chip	Apr-2014	7,080	40nm	362,052	-	$5.230e^{-03}$
Avalon A3222 Chip	Sep-2014	25,000	28nm	362,052	-	$1.850e^{-02}$
Avalon A6	Nov-2015	3,500,000	-	9,727	$9.320e^{-02}$	$6.940e^{-02}$
From 2016/01/01-2016/12/31						
Avalon A6	Nov-2015	3,500,000	-	72,836	-	$1.520e^{-02}$
Avalon A721	Nov-2016	6,000,000	-	20,918	$2.320e^{-01}$	$7.490e^{-02}$
From 2017/01/01-2017/04/30						
Avalon A721	Nov-2016	6,000,000	-	9,578	-	$2.860e^{-02}$
Avalon A741	Mar-2016	7,300,000	-	48,233	$2.460e^{-01}$	$1.750e^{-01}$

mining factories are generally located in remote areas. Finally, we assume that the efficiency of mining machines are quite similar to each other. This is a realistic assumption since the efficiency of mining equipment produced by the three largest companies during the same period is very close to each other.

The steps of simulation are listed as follows:

Step 1. Multiply the sales amount by the corresponding parameters to calculate the Hash power brought by each type of Avalon production (hardware and chips);

Step 2. The proportion of Hash power by Avalon production can be obtained from the data of whole Bitcoin network Hash power;

Step 3. The revenue from Avalon production equals the proportion obtained in above Step 2 times the revenue of the whole Bitcoin network;

Step 4. Since the price of mining equipment, transaction fees denominated in Bitcoin, and market price of Bitcoin are public information, then the equilibrium number of Bitcoin miners can be solved out from equation (4.12).

The number of mining equipment obtained from the simulation is summarized in following Table 4.3 and compared with the information of mining equipment number from Canaan IPO white paper.

Table 4.3 Comparison between Actual and Simulated Selling Amount

Product	Actual Selling Amount	Simulated Selling Amount
Avalon A3256 Chip	40,228	12,223
Avalon A3255 Chip	40,228	28,033
Avalon A3233 Chip	362,052	301,464
Avalon A3222 Chip	362,052	371,871
Avalon A6 (2015)	9,727	9,140
Avalon A6 (2016)	72,836	54,346
Avalon A721 (2016)	20,918	33,477
Avalon A721 (2017)	9,578	6,834
Avalon A741 (2017)	48,233	40,892

From the comparison of the two columns in Table 4.3, we can find that there exists a certain gap between the number of miners obtained from simulation and the actual number of mining machines. In order to investigate whether significant correlation exists between the simulated values and the actual values, a linear regression is performed. The estimation results are summarized in Table 4.4.

Besides, Figure 4.4 exhibits the actual, simulated and predicted selling amount of "Avalon" products. The estimation results in Table 4.4 show that the estimated slope is statistically significant while the estimated intercept is not. Therefore the simulated and predicted series can capture the trend of the historical data, although there exists deviation between the simulated and predicted values. The reason is that the available data is scarce, especially for the short term such as monthly data. As known, the aggregate Hash power of whole Bitcoin network grows very fast, which results in the bias of simulation using the annual data. So far, however, there does not exist any other related studies in the area to provide more precise simulation results.

Table 4.4 Regression of the Actual and Simulated Amount of Selling Amount

Regression Statistics					
Multiple R	9.890e-01				
R Square	9.780e-01				
Adjusted R Square	9.748e-01				
Standard Error	23134e-01				
Observation	9				
ANOVA					
	df	SS	MS	F	Significance F
Regression	1	1.663e11	1.663e11	310.641	4.664e-07
Residual	7	3.746e09	5.352e08		
Total	8	1.700e11			
	Coefficients	Standard Error	P-Value	Lower 95%	Upper 95%
Intercept	8271.914	9541.843	4.147e-01	-14290.958	30843.787
Slope	1.039	5.893e-02	4.664e-07	8.993e-01	1.178e-01

4.6 Conclusion

The fact that Bitcoin can only be obtained by calculation with high competition of electricity reflects its importance as an interesting issue on economic research. This paper constructs a benchmark mining model according to the Bitcoin protocol, and proposes an optimal Bitcoin mining strategy and the equilibrium level of investment. This benchmark model lays the foundation of the further research on the specific mining pattern.

Bitcoin block-chain is a public, distributed, and decentralized ledger. The winner of each round of mining competition is rewarded Bitcoin encrypted in the block and obtains the right to package the transactions into block and upload the block to Bitcoin block-chain. However, if Bitcoin miner does not win the competition, he or she will suffer a net loss. This net loss consists of two parts: the marginal cost, *i.e.*, the expenditure of electricity power; and the fixed cost in each mining period, *i.e.*, the investment on Bitcoin mining equipment.

Figure 4.4 Actual, Simulated & Predicted Selling Amount

The results show that for the Bitcoin miners, as long as a positive price exists, the optimal level of capital investment only depends on the comparison of his or her own marginal cost with that of other miners in the mining competition. Nevertheless, whether the winner will have a positive profit depends on the comparison between the revenue and his or her total cost. Moreover, the model shows that no matter how hard Bitcoin miners try to reduce their mining cost, the number of miners cannot be less than two if there existed more than one miner in the Bitcoin network initially, which prevents the appearance of monopoly at any subsequent mining periods. However, the prevention of monopoly does not guarantee that Hash power provided by each miner is less than 50%. Furthermore, this paper proposes a dynamic equilibrium in multiple stages of mining. Bitcoin miners seek the optimal capital investment at each period to maximize their total expected profit, subject to the total amount of input at each mining period cannot be greater than the sum of the initial endowment with the cumulative total profit (loss) till the previous period. The equilibrium strategy in this situation is the same as that in the static mining game.

Currently, the majority of Bitcoin miners are involved in the mining competition through the form of mining pool, which brings the challenges to simulate the equilibrium equations in the model. By using the data provided by "Canaan", the equilibrium equation of extreme case that all Bitcoin miners are homogeneous is simulated, and the evolution of Bitcoin miners has been approximated. Meanwhile, the difference between the simulated data and real data reflects the impact of the mining pool and monopoly of Bitcoin mining equipment in reality.

This work leads to promising future studies. First, the model of Bitcoin mining competition in this paper is based on Bitcoin protocol edited by [1]. However, in reality, the emergence of mining pools and professional mining equipment have set a barrier of entering mining competition. Therefore, a monopolistic competition market structure can be introduced to the model. Besides, the model assumes that the uncertainty of Bitcoin miners' expected returns is merely from the uncertainty of winning a competition. However, the market price is another source associated with the uncertainty. Bitcoin miners participate in the mining competition after referring to the exogenous market price of Bitcoin. Meanwhile, the market price may change after a miner wins the competition and provides the Bitcoins to trading platform. Therefore, the Bitcoin market pricing mechanism could be included in our models. Overall, our model provides a benchmark for Bitcoin mining competition in economics. Other features of Bitcoin mining industry can be extended in future research.

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CHAPTER 5. CONCLUSION

This dissertation fills the research gap in the application of microstructure theory on the RMB exchange rates and Bitcoin market price. A model of two RMB exchange rates determination is established in Chapter 2. Specifically, under a sequential trading mechanism, the participants in the free-floating offshore market make their trading decisions and determine the offshore RMB exchange rate with reference to the pre-determined onshore exchange rate. Then in the onshore managed floating exchange market, the commercial banks maximize their value function of foreign exchange transaction and the PBoC determines the official onshore exchange rate taking the reaction function of the commercial banks into account. The empirical analysis result suggests that the onshore and offshore exchange rates exhibit different patterns of long-term and short-term interactions before and after the "811 RMB exchange rate reform". Before the reform, the onshore exchange rate has greater impact on the offshore exchange rate, while after the reform, the offshore exchange rate becomes the leading indicator of the onshore exchange rate. In addition, the short-term causal relationship between the spot and the forward exchange rates evolves over time. Moreover, the foreign exchange reserve has significant impact on the exchange rate after the reform, which is the evidence of the intervention from the PBoC. Furthermore, not only the contemporaneous impact, but also the multi-period lag influence is detected between the two exchange rates.

A model of the market price of Bitcoin determination is established in Chapter 3. The participants in the Bitcoin trading market are classified as the fundamentalists and speculators. The different investment strategies and the interactions between the two types of participants determine the market price of Bitcoin in each period. The model brings two innovations to the microstructure theory. First, the fundamental value of Bitcoin system is defined as a function of the maturity of the technology and the degree of public acceptance. Second, the evolution of the speculator's sentiment is constructed as a nonlinear function of the overall speculator's opinion on the recent price trend. Besides, the empirical analysis confirms that both the historical and simulated return rates are characterized by long-term memory. The current price depends on its previous price, and it has persistent impact on the future prices. These features are consistent with that of classic speculative behavior in the financial market.

In Chapter 4, a benchmark model of Bitcoin mining competition is constructed based on the Bitcoin protocol, which laid a foundation for future research on specific mining patterns. The optimal Bitcoin mining strategy and the equilibrium level of input is proposed. As long as a positive market price of Bitcoin exists, the optimal input level of Bitcoin miners will depend on the comparison between their own marginal costs and that of other miners in the mining competition. Meanwhile, whether the winner in each round of competition has a positive profit or not depends on the comparison between its aggregate revenue and cost. Additionally, given there exist multiple miners in the Bitcoin network initially, there will be at least two miners in any sequential periods, no matter how Bitcoin miners attempt to reduce their mining costs. However, preventing monopoly does not guarantee that the proportion of hash power of each miner will be less than 50%.

In summary, the relationship among macro-economic variables can serve as a reasonable guide to study the long-term changes in asset prices. However, the traditional macroeconomic theory lacks the ability to interpret and predict the short-term changes in asset prices. This dissertation provides an alternative explanation for the persistent deviation of asset price from its fundamentals by applying microstructure theory to the RMB exchange rates and Bitcoin market price. For future research, an integrated approach to the synthesis of the theory of long-term asset price determination and the microstructure theory will be a promising research direction.